(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1247 Roll No. ☐☐☐☐☐☐☐☐☐☐

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2013-14

DISCRETE STRUCTURES

Time : 3 Hours   *Total Marks : 100*

Note :–Attempt all Sections.

SECTION–A

1. Attempt **all** parts :   (10×2=20)

   (a) Let A = {a, {a}}. Determine whether the following statements are true or false :

   (i)   {a, {a}} ∈ P(A)

   (ii) {a, {a}} ⊆ P(A)

   (iii) {{{a}}} ∈ P(A)

   (iv) {{{a}}} ⊆ P(A).

   (b) Find out the cardinality of the following sets :

   A = {x : x is weeks in a leap year}

   B = {x : x is a +ve divisor of 24 and not equal to zero}

   C = {{{}}}

   D = {{∅, {∅}}}.

(c) How many symmetric and reflexive binary relations are possible on a set S with cardinality n ?

(d) Define transitive closure with suitable example.

(e) Find the minimum number of students in a class to show that five of them are born on same month.

(f) Find the total number of squares in a chessboard.

(g) Define Group with suitable example.

(h) Define Lagrange's theorem. What is the use of the theorem ?

(i) Determine by means of truth table the validity of DeMorgan's theorem for three variables :

$(ABC)' = A' + B' + C'.$

(j) Define Binary Tree Traversal with example.

SECTION-B

2. Attempt all parts : (3×10=30)

(a) Let $(A, \leq)$ be a partially ordered set. Let $\leq$ be a binary relation on A such that for a and b in A, a is related to b iff $b \leq a$.

(i) Show that $\leq$ partially ordered relation.

(ii) Show that $(A, \leq)$ is lattice or not.

(b) (i) Define cyclic group with suitable example.

(ii) Simplify the following Boolean functions using three variable maps :

(a) $F(x, y, z) = \Sigma(0, 1, 5, 7)$

(b) $F(x, y, z) = \Sigma(1, 2, 3, 6, 7)$

(c) (i) Show that in a connected planner linear graph with 6 vertices and 12 edges, each of the regions is bounded by 3 edges.

(ii) Show that a regular binary tree has an odd number of vertices.

SECTION–C

3. Attempt all parts : $(5 \times 10 = 50)$

(a) Let A = {2, 3, 6, 12, 24, 36} and relation ≤ be such that 'x ≤ y' iff x divides y. Draw Hasse Diagram and find minimal and maximal elements.

(b) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5, and 7.

(c) Solve the following recurrence relation :

(i) $a_r - 7a_{r-1} + 10a_{r-2} = 0$, given that $a_0 = 0$ and $a_1 = 3$.

(ii) Given that $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 12$ satisfy the recurrence relation $a_r + C_1 a_{r-1} + C_2 a_{r-2} = 0$, determine a_r.

OR

Prove by using mathematical induction that :

$$7 + 77 + 777 + \ldots\ldots\ldots + 777\ldots7 = 7/81[10^{n+1} - 9n - 10],$$

for every n∈N.

(d) (i) Given that the value of $P \to \overline{Q}$ is true, can you determine the value of $P \vee (P \leftarrow \to Q)$.

(ii) Construct the truth table for the following statements :

$(P \to \overline{Q}) \to \overline{P}$

$P \leftarrow \to (\overline{P} \vee \overline{Q})$.

OR

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (4, 3), (2, 2), (2, 1), (3, 1)\}$ is a relation defined on A. Find Transitive closure of R using Warshall's algorithms.

(e) (i) Suppose G is a finite cycle-tree graph with at least one edge. Show that G has at least two vertices of degree 1.

(ii) Show that a connected graph with n vertices must have at least $(n-1)$ edges.