

**B. TECH.**  
**(SEM-III) THEORY EXAMINATION 2019-20**  
**DISCRETE STRUCTURES & THEORY OF LOGIC**

*Time: 3 Hours*

*Total Marks: 70*

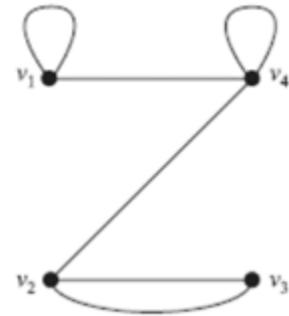
**Note:** 1. Attempt all Sections. If require any missing data; then choose suitably.  
2. Any special paper specific instruction.

**SECTION A**

**1. Attempt all questions in brief.**

**2 x 7 = 14**

- a. Draw all trees with exactly six vertices.
- b. Find the adjacency matrix  $A = [a_{ij}]$  of graph given in figure 1.
- c. Determine the power set  $P(A)$  of  $A = \{a, b, c, d\}$ .
- d. Define surjective function.
- e. Let  $f$  and  $g$  be the functions from the set of integers to the set of integers defined by  $f(x) = 2x + 3$  and  $g(x) = 3x + 2$ . What is the composition of  $f$  and  $g$ ? What is the composition of  $g$  and  $f$ ?
- f. Consider the following relations on  $\{1, 2, 3, 4\}$ :  
 $R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$ ,  
 $R_2 = \{(1, 1), (1, 2), (2, 1)\}$ ,  
 $R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$ ,  
 $R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$ ,  
 $R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$ ,  
 $R_6 = \{(3, 4)\}$ .  
 Which of these relations are reflexive?
- g. List all the ordered pairs in the relation  $R = \{(a, b) \mid a \text{ divides } b\}$  on the set  $\{1, 2, 3, 4, 5, 6\}$  and also display the graphical representation of the same.
- h. Find the values of the Boolean function represented by  $F(x, y, z) = xy + z'$ .



**Figure 1**

**SECTION B**

**2. Attempt any three of the following:**

**7 x 3 = 21**

a.	Prove the proposition $P(n)$ that the sum of the first $n$ positive integers is $\frac{1}{2}n(n + 1)$ ; that is, $P(n) = 1 + 2 + 3 + \dots + n = \frac{1}{2}n(n + 1)$																		
b.	Verify that the given propositions are tautology or not. i. $p \vee \neg(p \wedge q)$ ii. $\neg p \wedge q$																		
c.	Prerequisites in college is a familiar partial ordering of available classes. We write $A < B$ if course $A$ is a prerequisite for course $B$ . Let $C$ be the ordered set consisting of the mathematics courses and their prerequisites appearing in figure 2. <table style="margin-left: 20px; border: none;"> <thead> <tr> <th style="text-align: left; padding-right: 10px;">Class</th> <th style="text-align: left;">Prerequisites</th> </tr> </thead> <tbody> <tr><td>Math 101</td><td>None</td></tr> <tr><td>Math 201</td><td>Math 101</td></tr> <tr><td>Math 250</td><td>Math 101</td></tr> <tr><td>Math 251</td><td>Math 250</td></tr> <tr><td>Math 340</td><td>Math 201</td></tr> <tr><td>Math 341</td><td>Math 340</td></tr> <tr><td>Math 450</td><td>Math 201, Math 250</td></tr> <tr><td>Math 500</td><td>Math 450, Math 251</td></tr> </tbody> </table>	Class	Prerequisites	Math 101	None	Math 201	Math 101	Math 250	Math 101	Math 251	Math 250	Math 340	Math 201	Math 341	Math 340	Math 450	Math 201, Math 250	Math 500	Math 450, Math 251
Class	Prerequisites																		
Math 101	None																		
Math 201	Math 101																		
Math 250	Math 101																		
Math 251	Math 250																		
Math 340	Math 201																		
Math 341	Math 340																		
Math 450	Math 201, Math 250																		
Math 500	Math 450, Math 251																		
i.	Draw the Hasse diagram for the partial ordering $C$ of these classes.																		
ii.	Find all minimal and maximal elements of $C$ .																		
iii.	Does $C$ have a first element or a last element?																		

**Figure 2**

- d. What are the degrees and what are the neighborhoods of the vertices in the graphs  $G$  and  $H$  displayed in Figure 3?

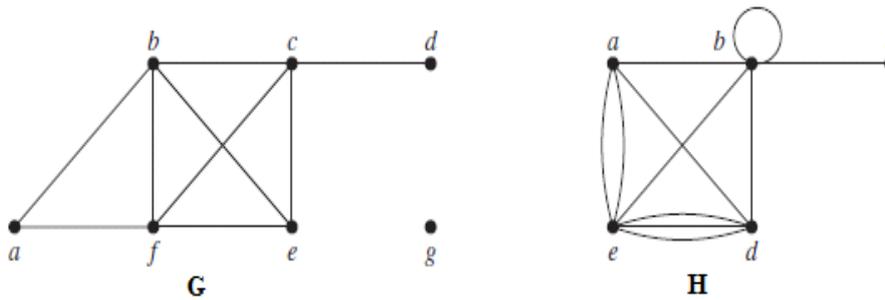


Figure 3

- e. For which values of  $n$  do these graphs have an Euler path but no Euler circuit?  
 i.  $K_n$  ii.  $C_n$  iii.  $W_n$  iv.  $Q_n$

**SECTION C**

3. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Answer these questions for the poset  $(\{3, 5, 9, 15, 24, 45\}, |)$ .  
 i. Find the maximal elements. ii. Find the minimal elements.  
 iii. Is there a greatest element? iv. Is there a least element?  
 v. Find all upper bounds of  $\{3, 5\}$ . vi. Find the least upper bound of  $\{3, 5\}$ .  
 vii. Find all lower bounds of  $\{15, 45\}$ . viii. Find the greatest lower bound of  $\{15, 45\}$ , if it exists.
- (b) Are the graphs  $G$  and  $H$  displayed in Figure 4 bipartite?

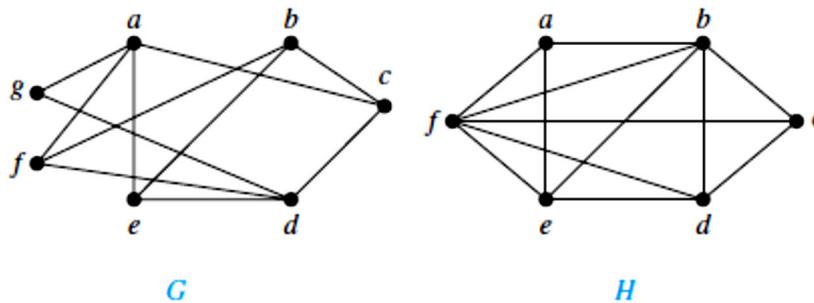


Figure 4

4. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Represent the expressions  $(x + xy) + (x/y)$  and  $xdx + ((xy + x)/y)$  using binary trees. Write these expressions in:  
 i. prefix notation.  
 ii. postfix notation.  
 iii. infix notation.
- (b) Construct the ordered rooted tree whose preorder traversal is  $a, b, f, c, g, h, i, d, e, j, k, l$ , where  $a$  has four children,  $c$  has three children,  $j$  has two children,  $b$  and  $e$  have one child each, and all other vertices are leaves.

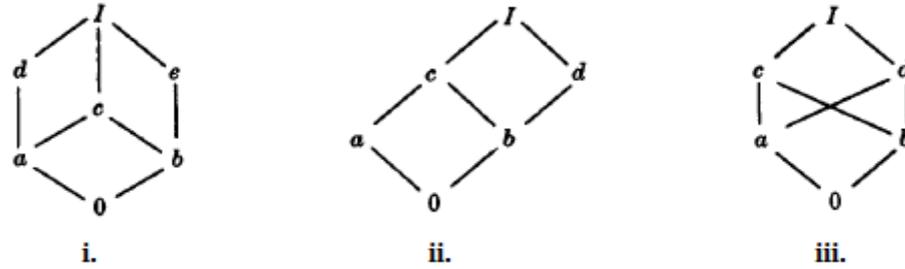
5. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Determine whether each of these statements is true or false.  
 i)  $0 \in \emptyset$       ii)  $\emptyset \in \{0\}$   
 iii)  $\{0\} \subset \emptyset$       iv)  $\emptyset \subset \{0\}$   
 v)  $\{0\} \in \{0\}$       vi)  $\{0\} \subset \{0\}$   
 vii)  $\{\emptyset\} \subseteq \{\emptyset\}$

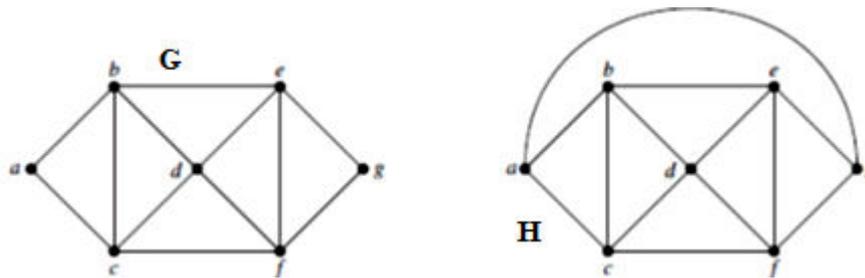
- (b) For each of these relations on the set  $\{1, 2, 3, 4\}$ , decidewhether it is reflexive, whether it is symmetric, whetherit is antisymmetric, and whether it is transitive.
- i)  $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
  - ii)  $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
  - iii)  $\{(2, 4), (4, 2)\}$
  - iv)  $\{(1, 2), (2, 3), (3, 4)\}$
  - v)  $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
  - vi)  $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

**6. Attempt any one part of the following: 7 x 1 = 7**

- (a) Which of the partially ordered sets in Fig 5 are lattices?



- (b) What are the chromatic numbers of the graphs  $G$  and  $H$  shown in Figure 6



**7. Attempt any one part of the following: 7 x 1 = 7**

- (a) Consider the group  $G = \{1, 2, 3, 4, 5, 6\}$  under multiplication modulo 7.
- (i) Find the multiplication table of  $G$ .
  - (ii) Find  $2^{-1}, 3^{-1}, 6^{-1}$ .
  - (iii) Find the orders and subgroups generated by 2 and 3.
  - (iv) Is  $G$  cyclic?
- (b) How many cards must be selected from a standard deck of 52 cards to guarantee that atleastthree cards of the same suit are chosen?