Printed Pages : 7 TCS-301

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1064 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. III) EXAMINATION, 2007-08

DISCRETE STRUCTURE

Time : 3 Hours] *[Total Marks : 100*

Note : *(1) Answer **all** questions.*

*(2) All questions carry **equal** marks.*

(3) In case of numerical problems assume data wherever not provided.

(4) Be precise in your answer.

1 Attempt any **four** of the following questions : 5×4=20

(a) Let $N = \{1, 2, 3, \ldots\}$ and a Relation is defined in $N \times N$ as follows (a, b) is related to (c, d) if and only if $ad = bc$ then show whether R is a equivalence relation or not.

(b) (i) Find the no. of partitions on $A = \{a, b, c, d\}$

(ii) Define symmetric difference and disjoint set.

(c) Draw the Hasse diagram of the relation s defined as |, "divides" on set B where $B = \{2, 3, 4, 6, 12, 36, 48\}$

(d) Let $A = \{1, 2, 3, 4, 5, 6\}$, construct pictorial discription of relation R on A for the following :

(i) $R = \{(J, K) \mid J \text{ is multiple of } K\}$

(ii) $R = \{(J, K) \mid (J - K)^2 \in A\}$

(iii) $R = \{(J, K) \mid (\sqrt{\ } \text{ divides } K)\}$

(iv) $R = \{(J, K) \mid J \times K \text{ is prime}\}$

(e) If $f : A \to B$ be both one to one and onto then $f^{-1} A \to B$ is both one to one and onto; prove the theorem.

(f) Prove that $(\sqrt{5})$ is not a rational number (prove by contradiction).

2 Attempt any **four** of the following questions : 5×4=20

(a) Simplify $F(A, B, C, D) = \sum(0, 1, 4, 5, 6, 8, 9, 12, 13, 14)$ using Karnaugh map.

(b) (i) Draw a logic circuit corresponding to Boolean expression $Y = \overline{A + BC} + B$

(ii) Differentiate between Multiplexer and Encoder.

(c) Show that $G = \{1, -1, i, -i\}$ where $i = \sqrt{-1}$, is an abelian group with respect to multiplication as a binary operation.

(d) Show that additive group z_4 is isomorphic to the multiplicative group of non zero element of z_5.

(e) Define permutation group. Let $A = \{1, 2, 3, 4, 5\}$. Find

$(1\ 3)\ 0\ (2\ 4\ 5)\ 0\ (2\ 3)$.

(f) (i) Differentiate between semigroup and subgroup with example.

(ii) Flow monoid, ring and field are related to each other.

3 Attempt any **two** of the following questions : **10×2=20**

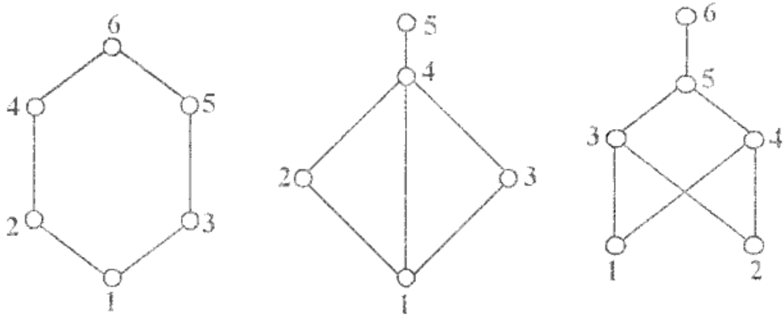
(a) (i) What do you mean by Graph Isomorphism, show it by example ?

(ii) Prove that if the graph has $'n'$ vertices and vertex $'u'$ is connected to vertex $'w'$ then there exists a path from $'u'$ to $'w'$ of length no more than $'n'$.

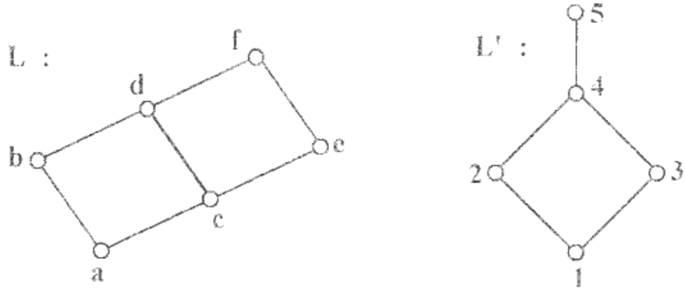
(b) (i) What is Binary Search Tree ? A binary search tree T and an ITEM of information is given. Write the algorithm which finds the location of ITEM in T or inserts ITEM as a new node in the tree.

(ii) Differentiate between Euler graph and Hamiltonian graph with examples.

(c) (i) Which of the partially ordered sets shown are lattices ?



(ii) Define isomorphic lattice. Show that the lattice L and L' given below are not isomorphic :



(d) (i) If $A=\{a, b, c\}$ prove that lattice $[P(A); \cup, \cap]$ (under $\subseteq$) is distributive when $P(A)=$ power set of A.

(ii) L is a bounded lattice. If L is distributive and the complement of an element $a \in L$ exists, then show that it is unique.

4 Attempt any **two** of the following questions : **10×2=20**

(a) (i) Show that

$((p \vee \sim q) \wedge (\sim p \vee \sim q)) \vee q$ is

a tautology using definition.

(ii) Define identity law, idempotent law absorption and involution law of logic.

(iii) What do you mean by logical equivalence ? Show one example of that.

(b) (i) Use quantifiers to say that $\sqrt{3}$ is not a rational number.

(ii) Let $M(x)$ be "x is mammal". Let $A(x)$ be "x is an animal", Let $W(x)$ be "x is warm blooded". Translate into a formula : Every mammal is warm blooded and translate into English :

$$(\exists x)\left(A(x) \wedge (\sim(M(x))\right)$$

(iii) Negate the proposition and show it by quantifiers : "All integers are greater than θ".

(c) (i) If truth set of any proportion over universal set is defined as $T_{p(n)} = \{a \in U / p(a) \text{ is true}\}$ then prove that

$$T_{p \leftrightarrow q} = \left(T_p \cap T_q\right) \cup \left(T_p^c \cap T_q\right)$$

$$T_{p \rightarrow q} = \left(T_p^c \cup T_q\right)$$

(ii) For all $n \geq 1, n^3 + 2n$ is a multiple of 3, prove it ?

(iii) Differentiate between Tautology and Contradiction.

5 Attempt any **two** of the following questions : **10×2=20**

(a) (i) How many selections any number at a time, may be made from three white balls, four green balls, one red ball and one black ball, if at least one must be chosen.

(ii) There are twelve students in the class. Find the number of ways that the twelve students take three different tests if four students are to take each test.

(b) (i) Determine the generating function of a numeric function a_r where

$$a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$$

(ii) Using generating function solve the recurrence relation

$$F(K) = F(K-2) + F(K-1) \quad F(0) = F(1) = 1$$

(c) What is Recursion and Recurrence Relation ? Solve the following recurrence relation using initial condition as

$$s(0) = s(1) = 1$$

$$s(k) - 9s(k-1) + 8s(k-2) = 9k + 1$$
