

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1064

Roll No.

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B.Tech.

THIRD SEMESTER EXAMINATION, 2006-07

DISCRETE STRUCTURE

Time : 3 Hours

Total Marks : 100

- Note :** (i) Attempt ALL questions.
(ii) All questions carry equal marks.
(iii) Be precise in your answer.

1. Attempt *any four* parts of the following : (5x4=20)
- (a) (i) Show that for any two sets A and B
 $A - (A \cap B) = A - B$.
- (ii) Give the power set of the set given below :
 $A = \{a, \{b\}\}$
- (b) (i) Let R be a binary relation defined as
 $R = \{\langle a, b \rangle \in \mathbb{R}^2 \mid a - b \leq 3\}$ determine whether R is reflexive, symmetric, anti symmetric and transitive.
- (ii) How many distinct binary relations are there on the finite set A ?
- (c) Let $X = \{1, 2, \dots, 7\}$ and
 $R = \{\langle x, y \rangle \mid x - y \text{ is divisible by } 3\}$
show that R is an equivalence relation.

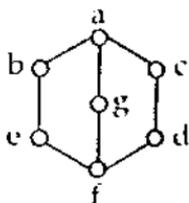
- (c) Define a group. Let $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $*$ denote "multiplication modulo 8 i.e. $x*y = (xy) \bmod 8$."

Write three distinct groups $(G, *)$ where $G \subset S$ and G has two elements.

- (d) What do you mean by group homomorphism and group isomorphism? Explain with example.
- (e) If $(R, +, \cdot)$ is a ring with unity, then show that, for all $a \in R$
- $(-1).a = -a$
 - $(-1).(-1) = 1$
- (f) Find the elements and the multiplication table of the symmetric groups S_3 .

3. Attempt *any four* parts of the following : (5x4=20)

- (a) Define Poset. Give an example of a set X such that $(P(X), \subseteq)$ is a totally ordered set.
- (b) Let A be a given finite set and $P(A)$ its power set. Let \subseteq be the inclusion relation on the elements of $P(A)$. Draw the Hasse diagrams of $(P(A), \subseteq)$ for $A = \{a, b, c\}$.
- (c) In the lattice defined by the Hasse given by the following figure :



How many complements does the elements 'e' have? Give all.

- (d) List all possible functions from $X = \{a, b, c\}$ to $Y = \{0, 1\}$ and indicate in each case whether the function is one to one, is onto and is one to one onto.
- (e) (i) Define an equivalence class generated by the elements of a set on a given equivalence relation.
- (ii) Let F_X be the set of all one to one onto mapping from X onto X , where $X = \{1, 2, 3\}$. Find all the elements of F_X and find the inverse of each element.
- (f) State and prove Pigeon hole principle.

2. Attempt *any four* parts of the following : (5x4=20)

- (a) Let $(A, *)$ be a semigroup, further more for every a and b in A , if $a \neq b$, then $a*b \neq b*a$.
- (i) Show that for every a in A
 $a*a = a$
- (ii) Show that for every a, b in A
 $a*b*a = a$
- (iii) Show that for every a, b, c in A
 $a*b*c = a*c$
- (b) Let G_1 and G_2 be sub group of a group G .
- (i) Show that $G_1 \cap G_2$ is also a subgroup of G .
- (ii) Is $G_1 \cup G_2$ always a subgroup of G ?

- (d) Define a boolean function. For any x and y in a boolean algebra show that $\overline{x \vee y} = \bar{x} \wedge \bar{y}$.
- (e) Write the following Boolean expressions in an equivalent product of sums canonical form in three variables x_1, x_2 and x_3 .
- (i) $x_1 * x_2$
- (ii) $x_1 \oplus x_2$
- (f) Define following terms :
- (i) Rooted tree
- (ii) Binary tree
- (iii) Binary search tree

4. Attempt *any two* parts of the following : (10x2=20)

- (a) (i) What is difference between conditional and biconditional statements ? Explain with example.
- (ii) Make a truth table for : $(P \rightarrow Q) \wedge (P \rightarrow R)$.
- (b) Show that the truth values of the following formulas are independent of their components.
- (i) $(P \wedge (P \rightarrow Q)) \rightarrow Q$
- (ii) $(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$
- (iii) $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
- (c) Show that given formula is a tautology
- $((P \vee Q) \wedge \neg (P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$

5. Attempt *any two* parts of the following : (10x2=20)

(a) Solve the following recurrence relations :

(i) $a_{n+1} - 1.5 a_n = 0, n \geq 0$

(ii) $a_n = 5a_{n-1} + 6a_{n-2}, n \geq 2, a_0 = a_1 = 3$

(b) Describe the 1-isomorphism and 2-isomorphism of the graph with example.

(c) Write short notes on any two of the following :

(i) Complete bipartite graph

(ii) Hamiltonian paths and circuit

(iii) Chromatic number of a graph

(iv) Euler graphs

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