(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1064 Roll No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

B.Tech.

THIRD SEMESTER EXAMINATION, 2005-2006

DISCRETE STRUCTURE

Time : 3 Hours *Total Marks : 100*

Note : *(i)* *Answer **ALL** questions.*

(ii) *Make suitable assumption for missing data, if any and state assumption made.*

(iii) *Be precise in your answer.*

1. Attempt *any four* of the following questions : **(5x4=20)**

(a) Draw a Venn diagram of sets A,B,C where -

(i) A and B have elements in common, B and C have elements in common, but A and C are disjoint

(ii) A⊆B, set A and C are disjoint, but B and C have elements in common.

(b) Let R be a reflexive and transitive relation on a set A. Define a new relation E on A as -

E = { <a,b> | <a,b> ∈ R ∧ <b,a> ∈ R }

Prove that E is an equivalence relation on A.

(c) Let X = {1, 2, 3, 4} and R = {<x,y> | x>y}

(i) Give the ordered pairs of R.

(ii) Draw the Graph of R

(iii) Give the relation matrix of R.

(d) Let f : R→R and g : R→R, where R is the set real numbers. Find fog and gof, where $f(x) = x^2 - 2$ and $g(x) = x + 4$. State where these functions are injective, surjective and bijective.

(e) What is meant by a recursively defined function ? Give the recursive definition of factorial function.

(f) State and prove the pigeon hole principle.

2. Attempt *any four* of the following questions : **(5x4=20)**

(a) Let ({p.q}, ∗) be a semigroup

Where p∗q = q, show that

(i) p∗q = q∗p

(ii) q∗q = q.

(b) Find all the sub groups of.

(i) $<Z_{12}, +_{12}>$

(ii) $<Z_5, +_5>$

(c) Define a group. Let S = {0, 1, 2, 3, 4, 5, 6, 7} and ∗ denote "multiplication modulo 8" that is $x * y = (xy) \bmod 8$

prove that ({0,1}, ∗) is not a group.

(d) Given a homomorphism f : G→G′; show that

(i) $f(e) = e^1$, where e and e^1 are the identity elements.

(ii) $f(a^{-1}) = (f(a))^{-1}$ for any element a in G.

(e) Show that the system (E, +, •) of even integers is a ring under ordinary addition and multiplication.

(f) Define the symmetric group of degree n, denoted by Sn,

3. Attempt *any four* of the following questions : **(5x4=20)**

(a) Define a Poset. Show that "less than or equal to" relation on set of real number is partial ordering.

(b) Let A be the set of factors of a particular positive integer m and let $\leq$ be the relation divides $\leq = \{<x,y> \mid x \epsilon A \wedge y \epsilon A \wedge (x \text{ divides } y)\}$
Draw Hasse diagram for m=30

(c) Show that the elements of the lattice $(N, \leq)$, where N is the set of positive integers and $a \leq b$ if and only if "a divides b" satisfy the destributive property.

(d) Define boolean function. for any x and y in a boolean algebra, show that $\overline{x \wedge y} = \bar{x} \vee \bar{y}$

(e) Write the following Boolean expressions in an equivalent sum of products canonical form in three variables x_1, x_2 and x_3.

(i) $x_1 * x_2$ (ii) $x_1 \oplus x_2$

(f) Prove that a binary tree with n nodes has exactly n+1 null branches.

4. Attempt *any two* of the following questions : **(10x2=20)**

(a) (i) Define the statement formula and well formed formula. Give some examples of each.

(ii) Construct the truth table of the following formula.

$$\urcorner(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee R))$$

(b) When a statement A is said to tautologically imply a statement B ? Verify the following implications

(i) $P \wedge (P \rightarrow Q) \Rightarrow Q$

(ii) $\urcorner Q \wedge (P \rightarrow Q) \Rightarrow \urcorner P$

(c) (i) Show that.

$((P \vee \urcorner Q) \wedge (\urcorner P \vee \urcorner Q)) \vee Q$ is a tautology.

(ii) Show that.

$(\urcorner P \wedge Q) \Rightarrow (Q \Rightarrow P)$ is not a tautology.

5. Attempt *any two* of the following questions : **(10x2=20)**

(a) (i) Solve the recurrence relation

$a_r - 2a_{r-1} - 3a_{r-2} = 0, \ r \geq 2$

by the generating function method with initial conditions $a_0 = 3$ and $a_1 = 1$.

(ii) Solve the following recurrence relation

$a_{n+1} - 1.5a_n = 0, \ n \geq 0.$

(b) (i) If G and $\overline{G}$ are bipartite, what can be concluded about G ?

(ii) Show that a Hamiltonian path is a spanning tree.

(c) Write short notes on any two of the following

(i) Eular Graphs.

(ii) Graph Coloring

(iii) Isomorphism and Homomorphism of graphs.

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