(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0111 Roll No. | | | | | | | | | | |

B. Tech.

(SEMESTER-III) THEORY EXAMINATION, 2012-13

DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours] *[Total Marks : 100*

Section – A

1. Attempt **all** questions. **2 × 10 = 20**

(a) Show that the set of all even numbers is a countably infinite set.

(b) Consider the following relations on the set A = {1, 2, 3, 4}, determine whether the following relations are reflexive, symmetric, anti-symmetric or transitive.

(i) R = {(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)}

(ii) R = A × A

(c) What is a composite relation ?

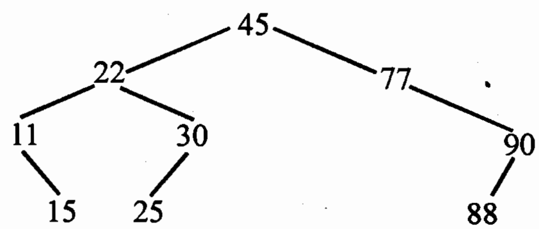
(d) Consider the set Z_4 = {0, 1, 2, 3}, together with multiplication modulo 4. Is Z_4 a group ?

(e) Consider statement 'p : If it rains the street get wet'. Consider statement 'q : The streets are wet.' Can we claim that if q is true then p is true' ?

(f) Is it necessary for a graph having Hamiltonian cycle to have an Eulerian tour ? Give proof of a counter-example.

(g) Draw a planar graph with 6 vertices with each vertex having degree more than or equal to 3.

(h) Write Pre-order and Post-order traversal of the following binary tree.



(i) What are the properties of a complemented Lattice ?

(j) Prove that the difference of some two integers out of (n + 1) integers divides n.

Section – B

Attempt **three** questions : **$10 \times 3 = 30$**

2. Attempt any **3** out of **5** :

(a) Let A, B, C be subsets of universal set U. Given that

$A \cap B = A \cap C$ and $\bar{A} \cap B = \bar{A} \cap C$

Is it necessary that B = C ? Justify your answer.

(b) What is a Partial Order ? Consider the set Z of integers. Define 'a R b' by '$b = a^r$' for some positive integer r. Show that R is a partial order on Z.

(c) Prove Lagrange's theorem that states "for any finite group G, the order (number of elements) of every subgroup H of G divides the order of G".

(d) What is a Tautology and what is Contradiction ? Draw truth tables for $\bar{p} \vee q$, $p \rightarrow q$, $p \leftrightarrow q$, $\bar{p} \vee \bar{q}$. Which of them are/is tautology ?

(e) Prove that : Let $(P, \leq)$ be a partially ordered set. Suppose the length of the longest anti-chain in P is n. Then the elements in P can be partitioned into n disjoint chains.

Section – C

Attempt **all** questions. In each question attempt either part 'A' or part 'B'. : **$10 \times 5 = 50$**

3. Part A : (1) A set S consists of two types of elements : Type 1 and Type 2. Both Type 1 and Type 2 subsets are non-empty. A relation R on S is defined such that $(a, b) \in R$ only if a and b are of different types. Can such a relation be reflexive, symmetric, transitive, equivalence ? Explain your answer.

(2) Prove by induction that the partial sum of the terms of Fibonacci sequence is

$F_0 + F_1 + F_2 + F_3 + \ldots + F_n = F_{n+2} - 1$

In a Fibonacci sequence every element is sum of previous two elements, with first two elements as 0 and 1. The Fibonacci sequence is given as 0 1 1 2 3 5 8 13 21 34...

OR

Part B : (1) Prove or disprove : $P(A \cup B) = P(A) \cup P(B)$, where P(X) denotes the power set of set X. A power set of X is a set containing all subsets of X.

(2) (i) Among the integers from 1 to 300, how many are not divisible by 2, nor by 3, nor by 5.

(ii) Among the integers from 1 to 300, how many are divisible by 2 and 3 but not by 5.

4. Part A : Consider the set $S = \{1, \omega, \omega^2\}$, where ω, ω^2 are complex cube roots of unity. If $\ast$ denotes the multiplication operation, show that the algebraic structure $(S, \ast)$ forms an Abelian group.

OR

Part B : Prove that a subgroup of a cyclic group is cyclic.

5. Part A : (a) Show that if the sum of the degrees for each pair of vertices of a graph G with n vertices is n or larger, then there exists a Hamiltonain cycle in G.

OR

Part B : (1) Let G be a disconnected graph with k connected components. How many minimum number of edges need to be added to make G connected ?

(2) Show that if all the internal vertices of a tree have same degree, then it has total odd number of vertices or give a counter-example.

6. Part A : (1) Let set S = {a, b, c, d, e} and set P be set of partitions of S such that P = {P1, P2, P3, P4}.

$P_1 = \{(a, b, c), (d, e)\}$, $P_2 = \{(a, b), (c, d, e)\}$, $P_3 = \{(a, b, c, d, e)\}$ and

$P_4 = \{(a), (b), (c), (d), (e)\}$.

A partial order on P is defined such that $P_I \leq P_J$ if and only if all elements of P_I are subset of elements of P_J.

(i) Express the partial order using a Hasse diagram.

(ii) Is this partial order a lattice ? Explain your answer.

(2) Prove that in a Boolean Lattice $(b \leq a)$ if and only if $(\bar{a} \leq \bar{b})$.

OR

Part B : (1) Let $(A, \leq)$ be a distributive lattice. Show that, if

$a \wedge x = a \wedge y$ and $a \vee x = a \vee y$

for some a, then $x = y$.

(2) Let a, b, c be elements in a non-distributive lattice $(A, \leq)$. Show that if $a \leq b$, then $a \vee (b \wedge c) \leq b \wedge (a \vee c)$.

7. Part A : (1) Given the premise show the truth or falsity of the following conclusion by drawing the truth table.

Premise :
1. If it rains, streets get wet
2. If streets get wet, Accidents happen
3. It has not rained

Conclusion : Streets are not wet.

(2) Prove if $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$?

OR

Part B : (1) Prove the validity of the following argument :

If I study, then I will not fail in Mathematics.

If I do not play Basketball, then I will study.

I failed in Mathematics.

I must have played Basketball.

(2) Prove if $\overline{p} \rightarrow q$ is equivalent to $\overline{q} \rightarrow p$?