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ECS303

B. Tech

(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10
DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours]   *[Total Marks : 100*

Note :   (i)   *Answer all questions.*

(ii)   *All questions carry equal marks.*

1   Attempt any **four** parts :   5×4=20

(a)   Let A be a set with 10 distinct elements. Determine the following :

(i)   Number of different binary relations on A

(ii)   Number of different symmetric binary relations on A

(b)   Suppose S and T are two sets and f is a functions from S to T. Let R_1 be an equivalence relation on T. Let R_2 be a binary relation on S such that $(x, y) \in R_2$ if and only if $(f(x), f(y)) \in R_1$. Show that R_2 is also an equivalence relation.

JJ-0111]   1   [Contd...

(c) Prove that union of two countably infinite set is countably infinite.

(d) Composition of functions is commutative. Prove the statement or give counter example.

(e) Prove that for any integer $n > 1$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots\ldots + \frac{1}{\sqrt{n}} > \sqrt{n}.$$

(f) What do you understand by asymptotic behaviour of a numeric function ? Explain Big-Oh and Big-Omega notation.

2 Attempt any **two** parts : 10×2=2

(a) (i) Define group. Prove that if every element of a group G is its own inverse then G is abelian group.

(ii) Prove that $(\mathbb{Z}_p, X_p)$ is a group where the set $\mathbb{Z}_p$ is the set of all non-zero residue classes modulo p, p is a positive prime number and X_P denotes the multiplication of residue classes modulo p.

(b) (i) Prove the following or give counter example :

If $(H_1, *)$ and $(H_2, *)$ are both subgroups of the group $(G, *)$ then $(H_1 \cup H_2, *)$ is also a subgroup of $(G, *)$.

(ii) State and prove Lagrange theorem.

(c) (i) Define and explain the following with suitable example :

(a) Cyclic group

(b) Zero divisor of a ring

(c) Order of an element of a group

(d) Field.

(ii) If G is a group of order n then order of any element $a \in G$ is a factor of n. Prove.

Answer any **two** parts : $10 \times 2 = 20$

(a) (i) Define a relation R on the set $\mathbb{Z}$ of all integers by $m R_n$ if and only if $m^2 = n^2$. Determine whether R is a partial order or not ?

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(a) (i) Define a relation R on the set $\mathbb{Z}$ of all integers by mR_n if and only if $m^2 = n^2$. Determine whether R is a partial order or not ?

(ii) Let $(A, \leq)$ and $(B, \leq)$ be two posets. Prove that $(A \times B, \leq)$ is a poset, where $(a, b) \leq (c, d)$ if and only if $a \leq c,\ b \leq d$.

(iii) Draw the Hasse diagram of $(A, \leq)$ where

$$A = \{3,\ 4,\ 12,\ 24,\ 48,\ 72\}$$

and the relation $\leq$ be such that $a \leq b$ if a divides b.

(b) (i) Define distributive lattice and complemented lattice. Prove that in a distributive lattice, if an element has a complement then this complement is unique.

(ii) Let

$$E(x_1,\ x_2,\ x_3) = \overline{\overline{(x_1 \vee x_2)} \vee (\bar{x}_1 \wedge x_3)}$$

be a boolean expression over the two valued boolean algebra. Write $E(x_1,\ x_2,\ x_3)$ in disjunctive normal form.

(c) (i) Let a, b, c be elements in a lattice $(A, \leq)$. Show that if $a \leq b$ then

$a \vee (b \wedge c) \leq b \wedge (a \vee c)$.

(ii) Simplify boolean function F given by

$F(A, B, C, D) = \sum (0, 2, 7, 8, 10, 15)$

using Karnaugh map.

4   Answer any **two** parts :

(a) (i) Given that the value of $p \rightarrow q$ is false, determine the value of $(\bar{p} \vee \bar{q}) \rightarrow q$.

(ii) Find a formula A that uses the variable p, q and r such that A is a contradiction.

(iii) Write an equivalent formula for

$p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$

which neither contains biconditional nor conditional connectives.

(iv) The contrapositive of a statement
S is given as "If $x < 2$ then
$x + 4 < 6$". write the statement S
and its converse.

(b) (i) Prove that $(p \lor q) \Rightarrow (p \land q)$ is logically
equivalent to $p \Leftrightarrow q$.

(ii) Translate the following sentences
in quantified expressions of
predicate logic.

(a) all students need financial aid.

(b) Some students need financial aid.

(c) (i) Show that following are not
equivalent :

(a) $\forall x (P(x) \rightarrow Q(x))$ and
$\forall x P(x) \rightarrow \forall x Q(x)$

(b) $\forall x \exists y P(x, y)$ and
$\exists y \forall x P(x, y)$.

(ii) Show that
$r \rightarrow \sim q, \ r \lor s, \ s \rightarrow \sim q, \ p \rightarrow q$
$\Leftrightarrow \sim p$ are inconsistent.

5   Answer any **two** parts :

(a) (i)   Find the simple expression for the generating
function of following discrete numeric
function

$$1, \frac{2}{3}, \frac{3}{9}, \frac{4}{27}, \ldots, \frac{(r+1)}{3^r}, \ldots$$

(ii)   Solve the recurrence relation

$$a_r - 6\,a_{r-1} + 8\,a_{r-2} = r \cdot 4^r$$

given $a_0 = 8,\ a_1 = 22$.

(b) (i)   Find the number of integer
solutions of the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 30$$

where $x_1 \geq 2,\ x_2 \geq 3,\ x_3 \geq 4,$
$x_4 \geq 2,\ x_5 \geq 0$

(ii)   Given the in order and post order traversal of
a tree T

In order : BEHFACDGI
Post order : HFEABIGDC
Determine the tree T and its pre
order.

(c) (i) Prove that for any connected planar graph,

$$V - e + r = 2$$

where v, e, r are the number of vertices, edges and regions of the graph respectively.

(ii) Define and explain the following :

(a) Bipartite graph

(b) Chromatic number of a graph

(c) Binary search tree

(d) Adjacency matrix of a graph.