(Following Paper ID and Roll No. to be filled in your Answer Books)

Paper ID : 2289953 **Roll No.** | | | | | | | | | | |

B.TECH

Regular Theory Examination (Odd Sem-III) 2016-17

COMPUTER BASED NUMERICAL AND STATISTICAL TECHNIQUES

Time : 3 Hours *Max. Marks : 100*

Note: Attempt all Sections. If require any missing data; then choose suitably.

Section - A

1. Attempt all questions in brief. (10×2=20)

a) Discuss the significant digits with suitable example.

b) The error in the measurement of the area of a circle is not allowed to exceed 0.1%. How accurately should the diameter be measured?

c) Define testing of Statistical hypothesis.

d) Express $1+x-x^2+x^3$ as sum of Chebyshev polynomial.

e) What is the condition of natural spline.

f) Write the normal equation for a $y=a+bx+cx^2$

g) Write a short note on floating point arithmetic.

h) Prove that $\mu\delta=\frac{1}{2}(\Delta+\nabla)=\frac{\Delta E^{-1}}{2}+\frac{\Delta}{2}$

i) Determine the condition number of the matrix $\begin{bmatrix}2 & -1 & 1\\ 1 & 0 & 1\\ 3 & -1 & 4\end{bmatrix}$ using the maximum absolute row sum norm.

j) Differentiate between ill conditioned and well conditioned methods.

Section - B

2. Attempt three questions from this section (3×10=30)

a) Use synthetic division and perform two interations for the Birge-Vieta method to find the smallest positive root of the equation

$x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$. Use the initial approximation $P_0 = 0.5$.

b) Write down the computer algorithms of least square curve fitting.

c) Derive the formula for error analysis of trapezoidal rule. If $I = \int_0^1 e^{-x^2} dx$, then estimate I using the Trapezoidal rule with the 10 subintervals. Find an error bound also.

d) Use Gauss-Elimination method to solve the following system of equations:

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

e) Use secant method to determine the root of the equation $\cos x - xe^x = 0$. Choose suitable initial approximation.

Section - C

3. Attempt any one part of the following: (1×10=10)

a) Find the condition for convergence of fixed point interation method. Find by fixed point iteration method, the real root of the equation $\sin x = 10(x-1)$.

b) Define Aitken's Δ^2 method. Find a real root of the equation $2x - \log_{10} x = 7$, correct to three decimal places using Aitken's Δ^2 method and iteration method. Also show how the rate of convergence of Aitken's Δ^2 method is rapid than iteration method.

4. Attempt any one part of the following: (1×10=10)

a) Write the algorithm for Lagrange's interpolation formula. Determine the step size that can be used in the tabulation of $f(x) = \sin x$ in the interval $[0, \pi/4]$ at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than 5×10^{-8}

b) Obtain an approximation in the sense of the principle of least squares in the form of a polynomial of the degree 2 to the function $1/(1+x^2)$ in the range $-1 \le x \le 1$.

5. Attempt any one part of the following: (1×10=10)

a) Calculate $y'(0.398)$ as accurately as possible using the table below and with the aid of the approximation S(h). Give the error estimate (the values in the table are correctly rounded.)

X:	0.398	0.399	0.400	0.401	0.402
f(x):	0.408591	0.409671	0.410752	0.411834	0.412915

b) Find a quadrature formula

$$\int_0^1 \frac{f(x)\,dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$$ which is exact for polynomials of highest possible degree.

Then use the formula on $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ and compare with the exact value.

6. Attempt any one part of the following: (1×10=10)

a) Apply Runge-Kutta method to find an approximate value of y for x = 0.2 and x = 0.4 if $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1

b) Solve by successive over relaxation method, the equations.

$$10x - 2y - 2z = 6$$
$$-x + 10y - 2z = 7$$
$$-x - y + 10z = 8$$

7. Attempt any one part of the following: (1×10=10)

a) Evaluate

$I = \int_0^1 \frac{dx}{2x^2 + 2x + 1}$, using the Lobatto 3 point and Radau 3-point formula. Compare with the exact solution.

b) i) A random sample of 900 members has a mean 3.4 cms. Can it be reasonably regarded as a sample from a large population of mean 3.2 cms and S.D. 2.3 cms.

ii) Find a uniform polynomial approximation of degree four or less to e^x on $[-1, 1]$ using Lanczos economization with a tolerance of $\varepsilon = 0.02$
