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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9911 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. III) EXAMINATION, 2007-08

ADVANCED MATHEMATICS - II

Time : 3 Hours] *[Total Marks : 100*

*Note : (i) Attempt **all** questions.*
*(ii) All questions carry **equal** marks.*

1 Attempt any **four** of the following :

(a) Examine whether the following vector in R^4 is linearly independent :

$(4,1,2,-6)$, $(1,1,0,3)$, $(1,-1,0,2)$, $(-2,1,0,3)$.

(b) Solve the following system of equations by the Gauss elimination method :

$$4x_1+x_2+x_3=4$$
$$x_1+4x_2-2x_3=4$$
$$3x_1+2x_2-4x_3=6$$

(c) Verify Cayley-Hamilton theorem for the matrix

$$A=\begin{bmatrix}1 & 2 & 0\\ -1 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$$ and hence obtain A^{-1}.

(d) Find eigenvalues and eigenvectors of the matrix :

$$A=\begin{bmatrix}3 & 1 & -1\\ -2 & 1 & 2\\ 0 & 1 & 2\end{bmatrix}$$

Hence, find P such that $P^{-1}AP$ is a diagonal matrix.

(e) Obtain the symmetric matrix B for the quadratic form :

$$Q=x_1^2+2x_1x_2-4x_1x_3+6x_2x_3-5x_2^2+4x_3^2$$

(f) Prove that the eigenvalues of an Hermitian matrix are real.

2 Attempt any **four** of the following :

(a) Solve the differential equation

$$(2x-4y+5)\frac{dy}{dx}+x-2y+3=0.$$

(b) Solve the differential equation

$$\frac{dy}{dx}+y\tan x=\sin 2x.$$

(c) Solve the differential equation

$$x^2\frac{d^2y}{dx^2}-6y=0.$$

(d) Find a general solution of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=12e^x/x^3$.

e) Apply the power series method to solve the differential equation :

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$

(f) Show that

$$\frac{1}{\sqrt{1-2xu+u^2}}=\sum_{n=0}^{\infty}p_n(x)u^n.$$

3 Attempt any **three** parts of the following :

(a) Find the most general analytic function $f(z)$ whose real part is $u=x^2-y^2-x$.

(b) Integrate $g(z)=\dfrac{z^2+1}{z^2-1}$ in the counter clockwise sense around a circle of radius 1 with centre at the point $z=1$.

(c) Find the Laurent series of $f(z)=\dfrac{1}{1-z^2}$ that converges in the annulus $\dfrac{1}{4}<|z-1|<\dfrac{1}{2}$ and determine the precise region of convergence.

4 Attempt any **two** parts of the following :

(a) Find the Laplace tra nsform of $f(t)$, where $f(t)$ is a periodic function with period T.

(b) Using convolution theorem evaluate

$$L^{-1}\left[\frac{1}{s^2(s+1)^2}\right].$$

(c) Using Laplace transform solve

$$\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+3\frac{dy}{dx}-y=t^2e^t$$

$$y(0)=1$$

$$\frac{dy}{dx}(0)=0$$

$$\frac{d^2y}{dx^2}(0)=-2.$$

5 Attempt any **two** parts of the following :

(a) Find the Fourier series of the function

$$f(x)=\begin{cases}k, & -\frac{\pi}{2}<x<\pi/2\\ 0, & \pi/2<x<\frac{3\pi}{2}\end{cases}$$

hence show that

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots\ldots\ldots=\pi/4.$$

(b) Find the Fourier cosine integral of $f(x)=e^{-kx}$ $(x>0, k>0)$.

(c) Find the Fourier transform of e^{-at^2}, $a>0$.