V-9955

Printed Pages : 4 **BT-201**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9955 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. II) EXAMINATION, 2006-07

ADVANCED MATHEMATICS

Time : 3 Hours] *[Total Marks :* ***100***

Note : *Attempt all the questions. Internal choice is mentioned for each question.*

1 Attempt any **four** parts of the following : **5×4=20**

(a) Examine the convergence of the series

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+.....$$

(b) Test the convergence of the series

$$\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+\frac{7}{4\cdot 5\cdot 6}+.........$$

(c) Discuss the convergence / divergence of the series

$$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+.......+\frac{1}{n^p}+........,\quad 0\le p<\infty$$

(d) Test the nature of the series

$$1+\frac{1+\alpha}{1+\beta}+\frac{(1+2\alpha)(1+\alpha)}{(1+\beta)(1+2\beta)}+\frac{(1+\alpha)(1+2\alpha)(1+3\alpha)}{(1+\beta)(1+2\beta)(1+3\beta)}+......$$

(e) Discuss the convergence of Reries

$$x+\frac{2^2x^2}{\lfloor 2}+\frac{3^3x^3}{\lfloor 3}+\frac{4^4x^4}{\lfloor 4}+\frac{5^5x^5}{\lfloor 5}+.....$$

(f) Find the radius of convergence of the series

$$\frac{\alpha x}{2}+\frac{\alpha^2x^2}{5}+\frac{\alpha^3x^3}{10}+..........+\frac{\alpha^n x^n}{n^2+1}+........$$

2 Attempt any **four** parts of the following : **5×4=20**

(a) If $I_n = \int_0^{\pi/4} \tan^n x\, dx$ prove that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

(b) Evaluate $\int_0^a x^2(a^2-x^2)^{3/2}dx$.

(c) Find the area common to the ellipses $a^2x^2+b^2y^2=1$ and $b^2x^2+a^2y^2=1$.

(d) Find the total length of the curve $x^{2/3}+y^{2/3}=a^{2/3}$.

(e) Find the volume generated by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about major axis.

(f) Define the gamma and beta functions. Prove that

$$B(m,\ n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n},\ \ m > 0,\ n > 0$$

3 Attempt any **two** parts of following : **10×2=20**

(a) Differentiating $n+2$ times successively the function $y = e^{m \cos^{-1} x}$, prove that

$$(1+x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$$

(b) Using Lagrange multipliers, find the minimum and the maximum distances of the points $(3,\ 4,\ 12)$ from the sphere $x^2+y^2+z^2=1$.

(c) Evaluate the following integral by changing to polar coordinates

$$\int_0^a \int_y^a \frac{x^2\, dx\, dy}{\sqrt{x^2+y^2}}.$$

4 Attempt any **two** parts of the following : **10×2=20**

(a) Define gradient, divergence and curl. If $\vec{F}$ is vector function, prove that

(1) Cural Curl $\vec{F} = grad\ \ dw\ \vec{F} - \nabla^2\ \vec{F}$

(2) $div\, cul\ \ \vec{F} = 0$

(b) Find the work done by a $\vec{F}(x, y) = (x^2 + y^2)\hat{i} + e^{xy}\hat{j}$ applied at a point P (1, 1) to displace it to point $Q(2, 3)$.

(c) Verify the Gauss divergence theorem for $\bar{F} = 4zx\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$ taken over the cube bounded the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$

5 Attempt any **two** parts of the following : **10×2=20**

(a) Prove that Poisson distribution is the limiting case of binomial distribution.

(b) State Bayes theorem. Three machines M_1, M_2 and M_3 produce identical items. Of their respective output 5%, 4% and 3% of items are defective, on a certain day M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item is selected at random is found defective. What are the probabilities that it was produced by the machine with the highest output.

(c) If the probability of a bad reaction from a certain injection is 0.001, determine the probability that out of 10000 individuals more than two will get a bad reaction.