B TECH
(SEM I) THEORY EXAMINATION 2018-19
ELEMENTARY MATHEMATICS-I

Time: 3 Hours   *Total Marks: 100*

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1.   **Attempt *all* questions in brief.**   **2 x 10 = 20**

a.   Evaluate the given limit: $\lim\limits_{x \to 2} \dfrac{x^2 - 3x + 2}{x^2 - 5x + 6}$

b.   State Mean Value Theorem.

c.   Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$.

d.   If $y = x^3 \cos 2x$, find $\dfrac{dy}{dx}$

e.   Evaluate $\int_0^{\frac{\pi}{2}} x \cos x \, dx$.

f.   Find the degree and order of the differential equation $\dfrac{d^2 y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$

g.   If $P(A) = \dfrac{3}{5}$ and $P(B) = \dfrac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

h.   Find maximum value of $2x^3 - 24x + 107$ in the interval *[1, 3]*.

i.   Write the Bernoulli's theorem on Binomial distribution.

j.   Find the probability of drawing two kings from a pack of cards in two successive draws, a card not being replaced

SECTION B

2.   **Attempt any *three* of the following:**   **10x3=30**

a.   Differentiate $y = 2e^{3x} + \log(\sin x) + x^2 \tan x$.

b.   State Lagrange's Mean Value theorem, then verify it for the fun $f(x) = 2x^2 + 3x + 5$.

c.   If $y = A\sin x + B\cos x$ then prove that $\dfrac{d^2 y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$.

d.   Evaluate $\int \dfrac{1 + \sin x}{1 - \sin x} dx$.

e.   Two friends appear in an interview for two vacancies in the same post. The probability of one's selection is $\dfrac{1}{7}$ and that of other's selection is $\dfrac{1}{5}$. What is the probability that:

         i.   Both of them will be selected

        ii.   Only one of them will be selected

      iii.   None of them will be selected

      iv.   At least one of them will be selected

SECTION C

3.   **Attempt any *one* part of the following:**   **10x1=10**

a.   If $y = \left(\dfrac{1 - \tan x}{1 + \tan x} \right)$, show that $\dfrac{dy}{dx} = \dfrac{-2}{(1 + \sin 2x)}$.

b.   Find the slope of the following curve $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$ at the point (*-1, -1*) and (*3, 9*).

4.   **Attempt any *one* part of the following:**   **10x1=10**

a.   The length of a rectangle x is decreasing at the rate of *3cm/minute* and the width y is increasing at the rate of *2cm/minute*. Find the rates of change of the area and perimeter of the rectangle if *x = 10 cm* and *y = 6 cm*.

b.   Find the area bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$.

5.   **Attempt any *one* part of the following:**   **10x1=10**

a.   Discuss the application of Rolle's theorem to the function $f(x) = x^{\frac{2}{3}}$ in $[-1, 1]$.

b.   Find Maxima and Minima for the function $f(x) = x^3 - 3x^2 + 7$.

6.   **Attempt any *one* part of the following:**   **10x1=10**

a.   Evaluate $\int \dfrac{x^2 + x + 2}{(x - 2)(x - 1)} dx$.

b.   Show $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

7.   **Attempt any *one* part of the following:**   **10x1=10**

a.   What is a chance that a leap year selected at random will contain 53 Sundays?

b.   An urn contains 3 red and 4 black balls. Two balls are taken out at random. Find the probability that the balls are of (i) same colour and (ii) different colours.