

**B. TECH.**  
**(SEM-IV) THEORY EXAMINATION 2017-18**  
**MATHEMATICS - III**

**Time: 3 Hours****Total Marks: 70****Note:** Attempt all Sections. If require any missing data, then choose suitably.**SECTION A****1. Attempt all questions in brief.****2 x 7 = 14**

- a) Discuss Singularity and its types.
- b) Write Cauchy-Riemann equation in polar co-ordinates.
- c) The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are insured, how many pairs would be expected to need replacement after 12 months? Given that  $P(z \geq 2) = 0.0228$ .
- d) Determine moment generating function of Binomial distribution.
- e) Prove that:  $E^2 = \mu + \frac{1}{2} \delta$
- f) Write Newton-Cote's quadrature formula.
- g) Find Z-transform of  $f(k) = u(-k)$ .

**SECTION B****2. Attempt any three of the following:****7 x 3 = 21**

- a) Determine an analytic function  $f(z)$  in terms of  $z$  if  $u + v = 2 \frac{\sin 2x}{e^{2y}} + e^{2y} - 2 \cos 2x$ .
- b) Find the mean variance of Poisson distribution.
- c) Find  $\int_0^6 \frac{e^x}{1+x} dx$  using (i) Trapezoidal rule, (ii) Simpson's 1/3<sup>rd</sup> rule and (iii) Simpson's 3/8<sup>th</sup> rule.
- d) A rod is rotating in a plane. The following table gives the angle  $\theta$  (in radians) through which the rod has turned for various values of time  $t$  (in seconds).

t:	0	0.2	0.4	0.6	0.8	1.0	1.2
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$\theta$ :	0	0.12	0.49	1.12	2.02	3.20	4.67
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Calculate the angular velocity and angular acceleration at  $t = 0.2$  and  $t = 1.2$  second.

- e) Find Fourier cosine transform of  $\frac{1}{1+x^2}$ , hence find Fourier sine transform of  $\frac{1}{1+x^2}$

**SECTION C**

**3. Attempt any one part of the following:** **7 x 1 = 7**

- (a) Verify Cauchy theorem by integrating  $e^{iz}$  along the boundary of the triangle with the vertices at the points  $1+i, -1+i$  and  $-1-i$ .
- (b) Evaluate  $\int_0^{\infty} \frac{\sin mx}{x} dx, m > 0..$

**4. Attempt any one part of the following:** **7 x 1 = 7**

- (a) The following table represents the height of a batch of 100 students. Calculate skewness and kurtosis:

Height (in cm)	59	61	63	65	67	69	71	73	75
No. of students	0	2	6	20	40	20	8	2	2

- (b) Use the method of least squares to fit the curve  $y = \frac{c_0}{x} + c_1\sqrt{x}$  to the following table of values:

x	0.1	0.2	0.4	0.5	1	2
y	21	11	7	6	5	6

**5. Attempt any one part of the following:** **7 x 1 = 7**

- (a) Find the root of the equation  $xe^x = \cos x$  using Regula-Falsi method correct to four decimal places.
- (b) Find Newton's divided difference polynomial for the following data:
- |       |     |     |     |     |      |
|-------|-----|-----|-----|-----|------|
| x:    | -3  | -1  | 0   | 3   | 5    |
| f(x): | -30 | -22 | -12 | 330 | 3458 |

**6. Attempt any one part of the following:** **7 x 1 = 7**

- (a) Solve the initial value problem  $u' = -2tu^2, u(0) = 1$  with  $h = 0.2$  on the interval  $[0, 0.4]$ . Use Runge-Kutta fourth order method and compare your result with exact solution.
- (b) Solve the following system of linear equations by Matrix decomposition method taking  $l_{ii} = 1$  for  $1 \leq i \leq 3$ .

$$3x - y + 2z = 12; \quad x + 2y + 3z = 11; \quad 2x - 2y - z = 2$$

**7. Attempt any one part of the following:** **7 x 1 = 7**

- (a) Using Z-transform, solve the following difference equation:

$$y_{k+2} + 4y_{k+1} + 3y_k = 3^k, \text{ given that } y_0 = 0 \text{ and } y_1 = 1.$$

- (b) The temperature  $u$  in the semi-infinite rod  $0 \leq x < \infty$  is determined by the differential equation

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \text{ subject to conditions}$$

- (i)  $u = 0$  when  $t = 0, x \geq 0$     (ii)  $\frac{\partial u}{\partial x} = -\mu$  (a constant) when  $x = 0$  and  $t > 0$ , (iii)  $u(x, t)$  is bounded.

Determine the temperature  $u(x, t)$ .