

(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 3987**Roll No. 

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**B.Tech.**

(SEM IV) EVEN SEMESTER THEORY EXAMINATION, 2009-2010

**MATHEMATICS - III**

Time : 3 Hours

Total Marks : 100

- Note :** (i) Attempt **ALL** questions.  
(ii) Each question carries **equal** marks.

1. Attempt any four parts of the following :

(4x5=20)

(a) Show that the function  $u = \frac{1}{2} \log(x^2 + y^2)$  is harmonic and find its harmonic.(b) Using Cauchy's integral formula, evaluate  $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$  where  $c$  is  $|z|=3$ .(c) Expand  $f(z) = \frac{1}{(z-1)(z-2)}$  in Laurent's series valid for the regions :(i)  $1 < |z| < 2$ (ii)  $0 < |z-1| < 1$ (d) Using complex integration method, evaluate  $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$ ;(e) Use contour integration method to evaluate  $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx, a > 0$ .(f) Evaluate  $\int_0^{1+i} (x^2 - iy) dz$  along the path  $y=x$  and  $y=x^2$ .

2. Attempt any four parts of the following : (4x5=20)

(a) The equation  $f(x) = (x-1)^2 (x-3)^2$  has roots at  $x=1$  and  $x=3$ . Which of the following methods can be applied to find all the roots ?

- (i) Bisection method
- (ii) False - position method
- (iii) Newton - Raphson method

Justify your answer.

- (b) Prove that the Newton Raphson method is second order convergent.
- (c) Perform five iteration of false position method to compute the smallest positive root of the equation  $3x + \sin x - e^x = 0$ .
- (d) Obtain the value of  $f(3.5)$  from the following data :

|        |   |     |    |    |    |
|--------|---|-----|----|----|----|
| $x$    | 3 | 4   | 5  | 6  | 7  |
| $f(x)$ | 3 | 6.6 | 15 | 22 | 35 |

- (e) Use Newton's divided difference method to compute  $f(5.5)$  from the following data :

|        |   |    |    |   |   |
|--------|---|----|----|---|---|
| $x$    | 0 | 1  | 4  | 5 | 6 |
| $f(x)$ | 1 | 14 | 15 | 6 | 3 |

- (f) Find the missing terms of the following data :

|        |     |     |                |     |     |     |     |
|--------|-----|-----|----------------|-----|-----|-----|-----|
| $x$    | 1.0 | 1.5 | <del>2.0</del> | 2.5 | 3.0 | 3.5 | 4.0 |
| $f(x)$ | 6   | ?   | 10             | 20  | ?   | 15  | 5   |

3. Attempt any two parts of the following : (2x10=20)

- (a) Find  $f'(1.1)$  and  $f''(1.1)$  from the following table :

|        |     |       |       |       |       |       |
|--------|-----|-------|-------|-------|-------|-------|
| $x$    | 1.0 | 1.2   | 1.4   | 1.6   | 1.8   | 2.0   |
| $f(x)$ | 0.0 | 0.128 | 0.554 | 1.296 | 2.432 | 4.000 |

- (b) Derive the formula for Simpson's  $\frac{3}{8}$  rule and find the value of the integral

$$\int_0^1 \frac{dx}{1+x^2}$$

Taking 12 intervals.

- (c) Using Runge - Kutta fourth order method to solve the following differential

equation  $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$  with  $y(0) = 1.0$  at  $x = 0.2, 0.4$ .

4. Attempt any two parts of the following :

(2x10=20)

- (a) Fit a relation  $y = ax + \frac{b}{x}$  which satisfies the following data, using method of least squares :

|   |     |     |     |      |      |      |      |      |
|---|-----|-----|-----|------|------|------|------|------|
| x | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    |
| y | 5.4 | 6.2 | 8.2 | 10.3 | 12.6 | 14.8 | 17.2 | 19.5 |

$$y = a_n + b_n^2$$

$$y = a_n^2 + b_n$$

- (b) What do you mean by regression analysis, explain ? If for two random variables,  $x$  and  $y$  with the same mean, the two regression equations are  $y = ax + b$  and  $x = \alpha y + \beta$

show that  $\frac{b}{\beta} = \frac{1 - a}{1 - \alpha}$ .

Also find the common mean.

- (c) Let the random variable  $X$  assume the value ' $n$ ' with the probability law  $p(X = n) = pq^{n-1}$ ,  $n = 1, 2, 3, \dots$

Find the moment generating function and hence mean and variance.

5. Attempt any two parts of the following :

(2x10=20)

- (a) What is chi-square test ? A survey of 320 families with 5 children show the following distribution :

|                        |         |        |         |         |         |         |
|------------------------|---------|--------|---------|---------|---------|---------|
| Number of boys & girls | 5 boys  | 4 boys | 3 boys  | 2 boys  | 1 boy   | 0 boys  |
|                        | 0 girls | 1 girl | 2 girls | 3 girls | 4 girls | 5 girls |
| Number of families     | 18      | 56     | 110     | 88      | 40      | 8       |

(Given  $\chi_{0.05}^2 = 11.1$  for 5 d.f.)

Test the hypothesis that male and female births are equally probable.

- (b) Distinguish between the np-chart, p-chart and c-chart of quality control analysis. The average percentage of defectives in 27 samples of size 1500 each was found to be 13.7%. Construct p-chart and give your conclusion regarding quality control.
- (c) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of this distribution ?

(Note : Ask for the table of area under normal curve from c.s.)

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