Printed Pages: 7 **NAS-401/EAS-401**

(Following Paper ID and Roll No. to be filled in your Answer Books)

Paper ID : 199419 **Roll No.**

B. TECH.

Theory Examination (Semester-IV) 2015-16

ENGG MATHEMATICS-III

Time : 3 Hours ***Max. Marks : 100***

Section-A

1. Attempt all questions of this section. Each question carry equal marks. (2×10 = 20)

(a) Write the cauchy's Reimaun conditions in polar coordinates system.

(b) Write the statement of generalized cauchy's integral formula for n^{th} derivative of an analytic function at the point $Z = Z_0$.

(c) Find the Z – transform of $U_n = \{a^n\}$

(d) Write the normal equations to fit a curve $y = ax^2 + b$ by least square method.

(e) If covariance between x and y variable is 10 and the variance of x and y are respectively 16 and 9, find the coefficient of correlation.

(f) The regression equations calculated from a given set of observations for two random variable are

$x = -0.4y + 6.4$ and $y = -0.6x + 4.6$ calculate mean values of x and y.

(g) Write the Newton's Raphson iterative formula to find the value of $\sqrt{N}$.

(h) Find the missing data in the given table :

				3
$f(x)$	580	556	–	465

(i) If $f(n)$ is given in following table :

x	0	0.5	1
$f(x)$	1	0.8	0.5

then using trapezoidal rule, evaluate

$$\int_0^1 f(x)dx$$

(j) Find the third forward difference with the arguments 2, 4, 6, 8 of the function $f(x) = x^3 - 2x$

Section-B

2. Attempt any five questions from this section. (10×5 = 50)

(a) Find the Laurent series for the function

$$f(z) = \frac{z^2 + 9z - 18}{Z^3 - 9z}, Z \text{ is complex variable}$$

valid for the regions

(i) $0 < |z| < 3$ (ii) $|z| > 3$

(b) Using calculus of residue, evaluate the following integral

$$\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$$

(c) Find the inverse Fourier sine transform of $\frac{1}{x} e^{-as}$

(d) Using least square method, fit a second degree polynomial from the following data :

x	0	1	2	3	4	5	6	7	8
y	12.0	10.5	10.0	8.0	7.0	8.0	7.5	8.5	9.0

Also estimate y at $x = 6.5$

(3) P.T.O.

(e) For the following data, calculate the finite differences and obtain the forward and backward difference polynomials. Also interpolate at $x = 0.25$ and $x = 0.35$

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

(f) Construct the divided difference table for the data.

x	0.5	1.5	3.0	5.0	6.5	8.0
$f(x)$	1.62	5.87	31.0	131.0	282.12	521.0

Hence find the interpolating polynomial and an approximation to the value of $f(z)$.

(g) Solve the system of equations AX=B, where

$$A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

using the LU decdomposition method. Take all the diagonal elements of L as 1.

(4)

(h) Solve the initial value problem

$$\frac{dy}{dx} = -2xy^2, y(0) = 1$$

with h = 0.1 on the interval [0,0.3]. Use the fourth order Runge-Kutta method.

Section-C

Note: Attempt any two questions from this section. Each question carry equal marks. (15×2=30)

3. (a) Show that for the function give as -

$$f(z) = \begin{cases} \frac{2xy(x+iy)}{x^2+y^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

The C-R conditions are satisfied at origin but derivative of $f(z)$ at origin does not exist.

(b) Verify that the function on $4(xy) = xy$ is harmonic and find its conjugate harmonic function. Express $u+iv$ as an analytic function $f(z)$.

$u = x^2 - y^2 - y$

(c) Find the Fourier transform of Block function $f(t)$ of height 1 and duration a defined by

$$f(t) = \begin{cases} 1 & \text{for } |t| \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

4. (a) Using Z - tranform, solve the difference equation

$$u_{n+2} - 4u_{n+1} + 3u_n = 5^n$$

with $u_0 = u_1 = 1$

(b) The first four moments of a distribution about x = 4 are 1, 4, 10, 45. Comment on the skewness and Kurtosis of the distribution.

(c) For 10 observations on price (x) and supply (y) the following data were obtained

$\Sigma x = 130, \Sigma y = 220, \Sigma x^2 = 2288$

$\Sigma x^2 = 5506$ and $\Sigma_{xy} = 3467$

Obtain the two lines of regression.

5. (a) Find the root of the euqation $xe^x = 3$ by regula talsi method correct up to two decimal places in the interval (1, 1.5).

(b) Prove the following identities :

(i) $\left(\frac{\Delta^2}{E}\right)\mu_x \neq \frac{\Delta^2\mu_x}{E\mu_x}$

(ii) $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{E(e^x)}{\Delta^2 e^x} = e^x$

(c) The velocity v of a particle at distance s from a point on its path is given by the following table :

s (m.)	0	10	20	30	40	50	60
v (m./s.)	47	58	64	65	61	52	38

Estimate the time taken to travel 60m. Using Simpson's one-third rule.