Printed Pages—4 **TAS301**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9958	Roll No.										

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2012-13

MATHEMATICS—III

Time : 3 Hours *Total Marks : 100*

Note :—*(1) Attempt* ***all*** *questions.*

(2) All questions carry equal marks.

(3) The symbols have their usual meaning.

1. Attempt any **FOUR** parts of the following : **(5×4=20)**

(a) Using Fourier integral representation, show that

$$\int_0^\infty \frac{w \sin wx}{1+w^2}\,dw = \frac{\pi}{2} e^{-x}, \quad x > 0.$$

$2\cos\frac{84\pi}{9}$ and

(b) Express the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

as Fourier integral. Hence evaluate the value of

$$\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda}\,dx.$$

(c) Find Fourier cosine transform of the following function :

$$F(x) = \begin{cases} x & , \quad 0 < x < \frac{1}{2} \\ 1-x & , \quad \frac{1}{2} < x < 1 \\ 0 & , \quad x > 1 \end{cases}$$

(d) Use Fourier sine transform to solve the equation :

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

under the conditions :

(i) $u(0, t) = 0$

(ii) $u(x, 0) = e^{-x}$

(iii) $u(x, t)$ is bounded.

(e) Find the z-transform of $\sin \alpha k$, $k \geq 0$.

(f) Solve the difference equation using Z-transform

$$y_{K+1} - 2y_{k-1} = 0,\ k \geq 1,\ y(0) = 1.$$

2. Attempt any **FOUR** parts of the following : **(5×4=20)**

(a) Prove that an analytic function with constant modulus is constant.

(b) If $u(x, y) = x^2 - y^2$ and $v(x, y) = \frac{y}{x^2 + y^2}$, prove that both u and v satisfy Laplace's equation but are not harmonic conjugates.

(c) Determine the analytic function where real part is $e^{2x}(x \cos 2y - y \sin 2y)$.

(d) Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$, along the real axis from z = 0 to z = 2 and then along a line parallel to y-axis from z = 2 to z = 2 + i.

(e) Find the value of $\int_C \frac{2z^2 + z}{z^2 - 1}\, dz$, where C is the circle of unit radius with centre at z = 1.

(f) Use Cauchy integral formula to evaluate

$$\int_C \frac{e^{2z}\, dz}{(z-1)(z-2)},$$ where C is the circle $|z| = 3$.

3. Attempt any **TWO** parts of the following : **(10×2=20)**

(a) Using Taylor's theorem, show that

$$\log z = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} - \ldots\ldots,$$

where $|z - 1| < 1$.

(b) Evaluate

$$\int_C \frac{12z - 7}{(z-1)^2 (2z+3)}\, dz,$$

where C is the circle

(i) $|z| = 2$

(ii) $|z + i| = \sqrt{3}$.

(c) Find a bilinear transformation which maps the points 1, i, –1 of the z-plane into i, 0, –i of the w-plane respectively.

4. Attempt any **TWO** parts of the following : **(10×2=20)**

(a) Find the measures of Skewness and Kurtosis on the basis of moments for the following distribution :

x :	1	3	5	7	9
f :	1	4	6	4	1

(b) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are eligible :

Variance of x = 9

Regression equations :

$$8x - 10y + 66 = 0,$$

$$40x - 18y = 214.$$

What were :

(i) the mean value of x and y

(ii) the standard deviation of y and

(iii) the coefficient of correlation between x and y.

(c) The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3, out of 1000 taxi drivers.

Find approximately the number of drivers with

(i) no accidents

(ii) more than 3 accidents in a year.

5. Attempt any **TWO** parts of the following : **(10×2=20)**

(a) The corresponding values of x and y are given below :

x :	87	84	79	64	47	37
y :	292	283	270	235	197	181

Fit a parabola of the form $y = ax^2 + bx + c$. Also find the value of y for $x = 80$.

(b) Solve the bi-quadratic equation :

$$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0.$$

(c) Show that the roots of the cubic equation $x^3 - 3x + 1 = 0$ are :

$$2\cos\frac{2\pi}{9},$$