(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9958 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2010-11

MATHEMATICS—III

Time : 3 Hours *Total Marks : 100*

Note : (1) Attempt **all** questions.

(2) All questions carry equal marks.

(3) Provide table for area under normal curve.

1. Attempt any **two** parts of the following :— **(10×2=20)**

(a) (i) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$, $a > 0$.

(ii) Using Fourier cosine integral for $f(x) = e^{-Kx}$, prove that

$$\int_0^\infty \frac{\cos \lambda x}{K^2 + \lambda^2} d\lambda = \frac{\pi e^{-Kx}}{2K}, \quad x > 0, \; K > 0.$$

(b) (i) Find the Fourier transform of

$$f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}.$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

(ii) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $x > 0, t > 0$ subject to the boundary conditions $u(0, t) = 0$, $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ and $u(x, t)$ is bounded.

(c) Solve the following difference equation using Z-transform

$$y_{n+2} - 4y_{n+1} + 3y_n = 5^n.$$

2. Attempt any **four** parts of the following :— **(5×4=20)**

(a) State Cauchy-Riemann's equation. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann's equations are satisfied at that point.

(b) Discuss the analyticity of $f(z) = z\bar{z}$.

(c) If ϕ and ψ are functions of x and y satisfying Laplace's equation, show that s + it is analytic, where

$$s = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \text{ and } t = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}.$$

(d) State Cauchy's integral formula. Hence evaluate

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz, \text{ where } C : |z| = 3.$$

(e) Find the value of integral $\int_0^{1+i} (x - y - ix^2)\, dz$ along the real axis from z = 0 to z = 1 and then along a line parallel to imaginary axis from z = 1 to z = 1 + i.

(f) State and prove Cauchy's theorem. Hence evaluate

$$\int_C \frac{z^2 + 5z + 6}{z - 2} dz, \text{ where, } C : |z| = \frac{3}{2}.$$

3. Attempt any two parts of the following :— **(10×2=20)**

(a) Find the bilinear transformation which maps $z = 1, i, -1$ respectively onto $w = i, 0, -1$. Hence find the image of $|z| \le 1$ under this transformation.

(b) Evaluate $\int_0^{\pi} \frac{a\,d\theta}{1+2a^2-\cos 2\theta}$, using contour integration.

(c) State Cauchy's Residue theorem. Hence evaluate

$$\int_C \frac{z^2}{(z-1)^2 (z+2)} dz, \text{ where } C : |z| = \frac{5}{2} .$$

4. Attempt any two parts of the following :— **(10×2=20)**

(a) Define the coefficient of Skewness and Kurtosis. Find the measures of Skewness and Kurtosis on the basis of moments for the following distribution :

x :	1	3	5	7	9
y :	1	4	6	4	1

(b) (i) Find the moment generating function of the random variable x having the probability function given by

$$f(x) = \begin{cases} x, & \text{when } 0 \le x < 1 \\ 2-x, & \text{when } 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(ii) Assume the mean height of soldiers to be 68·22 inches with a variance of 10·8 inches square. How many soldiers in a regiment of 10,000 would you expect to be over 6 feet tall ?

(c) (i) If θ is the acute angle between the two regression lines in case of two variables x and y, show that

$$\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} .$$

Also, explain the significance of the formula when $r = 0$ and $r = \pm 1$.

(ii) Two lines of regression are given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ and $\sigma_x^2 = 12$. Calculate the mean values of x and y, the coefficient of correlation between x and y.

5. Attempt any **two** parts of the following :— **(10×2=20)**

(a) Fit a second degree parabola to the following data :

x	:	1	2	3	4	5
y	:	25	28	33	39	46

(b) Solve $x^3 - 3x^2 + 12x + 16 = 0$ using Cardon's method.

(c) Solve the equation $x^4 + 8x^3 + 9x^2 - 8x - 10 = 0$ using Descarte's method.