



Printed Pages : 4

TAS-301/TMA-301

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9958

Roll No.

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B. Tech.

(SEM. III) EXAMINATION, 2007-08

MATHEMATICS - III

Time : 3 Hours]

[Total Marks : 100

Note : (1) Attempt all questions.

(2) Marks are shown against each equation.

1 Attempt any **four** of the following : **5×4=20**

(a) Using Fourier integral representation, show that

$$\int_0^{\infty} \frac{\cos(\lambda x)}{(1+\lambda^2)} d\lambda = \frac{\pi}{2} e^{-x}, \quad (x > 0)$$

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{for } |x| \leq 1, \\ 0 & \text{for } |x| > 1 \end{cases}$$

Hence or otherwise evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$.



- (c) Find the Fourier sine transform of $(e^{-ax} | x)$, $a > 0$.
 (d) Using Fourier transform solve

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0; \quad V(x, 0) = f(x).$$

- (e) Find the Z -transform of

$$\cosh\left(\frac{n\pi}{2} + \theta\right).$$

- (f) Solve by Z -transform :

$$y_{n+2} - 4y_n = 0 \quad \text{given that } y_0 = 0, \quad y_1 = 2.$$

2 Attempt any **four** of the following : 5×4=20

- (a) If $f(z) = u(r, \theta) + iv(r, \theta)$ is analytic in a domain D , prove that

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}.$$

- (b) Prove that an analytic function with constant modulus is constant.
 (c) Determine an analytic function $f(z)$, in terms of z whose real part is $e^{-x} (x \sin y - y \cos y)$.

- (d) Evaluate $\int_C \frac{\sin(\pi z) + \cos(\pi z)}{(z-1)(z-2)} dz$ where C is the circle $|z| = 4$.

(e) Prove that if $f(z)$ is analytic and bounded in the entire complex plane then $f(z)$ is a constant.

(f) Evaluate : $\int_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ where C is $|z|=4$.

3 Attempt any **two** of the following : 10×2=20

(a) Expand $f(z) = 1 / \{(z-1)(z-2)\}$ in the region :

- (1) $|z| < 1$
- (2) $1 < |z| < 2$
- (3) $|z| > 2$

(b) Using residue theorem, evaluate

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)(x^2 + 4)}$$

(c) Find the bilinear transformation which maps $z = 1, i, -1$ onto the points $w = i, 0, -i$ respectively. Hence find (i) the image of $|z| < 1$.
(ii) the invariant points of the transformation.

4 Attempt any **two** of the following : 10×2=20

(a) In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as

$$4x - 5y + 33 = 0 \text{ and } 20x - 9y = 107$$

respectively. Calculate \bar{x} , \bar{y} and the coefficient of correlation between x and y .

- (b) In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours, estimate the number of bulbs likely to burn for
- (1) more than 2150 hours
 - (2) less than 1950 hours
 - (3) More than 1920 hours and but less than 2160 hours.
- (c) Find the mean of a binomial distribution. Also find the moment generating function of the binomial distribution about its mean.

5 Attempt any **two** of the following : 10×2=20

- (a) Solve $x^3 - 3x^2 + 12x + 16 = 0$.
- (b) By the method of least squares, find the straight line that best fits the following data :

x	1	2	3	4	5	6	7
y	14	27	40	55	68	77	85

- (c) Find the least squares fit of the form $y = a_0 + a_1 x^2$ to the following data :

x	-1	0	1	2
y	2	5	3	0

