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Printed Pages – 6

TAS -301 / MA-301
MA-301(O) / TCF-304

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9909 9958 9959	Roll No.	

HIRD

B.Tech.

THIRD SEMESTER EXAMINATION, 2005-2006

MATHEMATICS - III

Time : 3 Hours *Total Marks : 100*

Note : *(i)* *Attempt **ALL** questions.*
(ii) *All questions carry equal marks.*
(iii) *In case of numerical problems assume data wherever not provided.*
(iv) *Be precise in your answer.*

1. Attempt ***any four*** parts of the following : **(4x5=20)**

(a) Define an analytic function. Let Re $f(z)$ be constant for an analytic function $f(z)$, then show that $f(z)$ is constant.

(b) Construct the analytic function $f(z)$ whose real part is $u(x, y) = e^{-x}(x \sin y - y \cos y)$.

(c) Evaluate the following integral

$$\int_{1+i}^{2+i} (2x + iy + 1)\, dz$$

along the two paths

(i) $x = t+1,\ y = 2t^2 + 1$

(ii) The straight line joining $1-i$ and $2+i$.

(d) State and prove the Cauchy's Integral Formula.

(e) State and verify, Cauchy theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i$, $-1+i$ and $-1-i$.

(f) Find the first three terms of the Taylor's series expansion of $f(z)=\dfrac{1}{z^2+4}$ about $z=-i$. Find the region of convergence.

2. Attempt *any two* parts of the following : **(10x2=20)**

(a) Using complex variable technique evaluate the real integral $\int_0^{2\pi}\dfrac{\sin^2\theta}{5+4\cos 3\theta}\,d\theta$.

(b) Evaluate by using contour integration method $\int_0^{\infty}\dfrac{\cos ax}{x2+1}\,dx$, $a \geq 0$.

(c) (i) Expand $f(z) = \dfrac{1}{z^2-3z+2}$ in Laurent's series valid in the region $1<|z|<2$

(ii) Define a conformal mapping. Prove that an analytic function $f(z)$ ceases to be conformal at the points z_0, where $f'(z_0)=0$.

Note - Following Q. No. 3 to 5 are for New Syllabus only (TAS-301/MA-301)

3. Attempt *any two* parts of the following : **(10x2=20)**

(a) Define the Fourier transform.

(i) State and prove the modulation theorem for the Fourier transform.

(ii) State and prove the Parseval identity for the Fourier transform.

(d) State and prove the Cauchy's Integral Formula.

(e) State and verify, Cauchy theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i$, $-1+i$ and $-1-i$.

(f) Find the first three terms of the Taylor's series expansion of $f(z)=\frac{1}{z^2+4}$ about $z=-i$. Find the region of convergence.

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3. Attempt *any two* parts of the following : **(10x2=20)**

(a) Define the Fourier transform.

(i) State and prove the modulation theorem for the Fourier transform.

(ii) State and prove the Parseval identity for the Fourier transform.

(b) Define the Z-transform of a sequence $\{f_k\}$ n=0

$$y_{k+2} = \frac{5}{6} y_{k+1} - \frac{1}{6} y_k + 3^k, y_1 = 1, y_0 = 0.$$

4. Attempt *any four* parts of the following : **(4x5=20)**

(a) Calculate the variance and third central moment from the following data :

xi	0	1	2	3	4	5	6	7	8
fi	1	9	26	59	72	52	29	7	1

(b) Find the moment generating function of the exponential distribution.

$$f(x) = \frac{1}{c} e^{-x/c}, 0 \le x \le \infty, c > 0.$$

(c) Define the lines of regression and coefficient of correlation. The ages of the husbands and wives are given in the following table

age of husband	x	23	27	28	29	30
age of wife	y	18	22	23	24	25

calculate the coefficient of correlation between x and y from the above table.

(d) Find the mean and variance of Poisson's Distribution.

(e) Assuming the half the population are consumers of chocolate so that the chance of an individual being a consumer is $\frac{1}{2}$, and assuming that 100 investigators each take 10 individuals to see whether they are consumers, how many investigators would you expect to report that three people or less were consumers ?

(f) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given

that if $f(t)=\frac{1}{\sqrt{2\pi}}\int_0^t e^{-\frac{1}{2}x^2}dx$

then $f(0.5) = 0.19$ and $f(1.4) = 0.42$.

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Define the skewness and kurtosis of a distribution. The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98 calculate the moments about the mean. Also evaluate β1, β2 and comment upon the skewness and kurtois of the distribution.

(b) Solve the biquadratic equation by Ferrari's method $x^4-8x^3-12x^2+60x+63=0$.

(c) The corresponding values of x and y are given below :

x :	87	84	79	64	47	37
y :	292	283	270	235	197	181

Fit a parabola of the form $y = ax^2+bx+c$. Also find the value of y for $x=80$ correct up to third place of decimal.

Note - Following Q.No. 3-5 are for old Syllabus (MA-301(O))

3. Attempt *any two* parts of the following : **(10x2=20)**

(a) A cantilever beam of length l and weighing w kg per unit length is subjected to a horizontal compressive force p applied at the free end. Taking the origin at the free end and y-axis upwards, establish the differential equation of the beam and hence find its maximum deflection.

(b) Solve the following differential equation in series about $x=0$

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$$

(c) Using z-transform, solve the following difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n, y_0=0, y_1=1$

4. Attempt *any four* parts of the following : **(4x5=20)**

(a) Using Rodrigue formula for Legender function

prove that $\int_{-1}^{1} x^m p_n(x)\,dx = 0$.

Where m and n are positive integers and $m<n$.

(b) Prove that $J_0^2+2J_1^2+2J_2^2 + \text{———} = 2$

(c) Prove that $\dfrac{1-t^2}{1-2tx+t^2} = T_0(x)+2\sum_{n-1}^{\infty} T_n(x)\,t^n$

Where $To(x)$ and $Tn(x)$ are chebyshev Polynomials, n is non-negative integer.

(d) Find $H^{-1}\{\bar{f}(p)\}$ where $\bar{f}(p) = \dfrac{e^{-ap}}{p}$, $\bar{f}(p)$ is the Hankel transform of $f(x)$ and $n=0$.

(e) Find the Fourier sine and cosine transform of $f(x) = x^{n-1}$

(f) Use Fourier transform to solve $\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}$

(i) $u=0$, when $x=0$, $t>0$

(ii) $u=\begin{cases}1, & 0<x<1\\ 0, & x\geq 1\end{cases}$ when $t=0$

(ii) $u(x,t)$ is bounded

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Solve the partial differential equation

$$(D^2 - DD^1 - 2D^{12} + 2D + 2D^1)z = e^{2x+3y} + \sin(2x+y)$$

(b) Use separation of variable method to solve the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Subject to the boundary conditions

$u\ (0, y) = u\ (\ell, y) = u\ (x, 0) = 0$ and

$$u(x, a) = \sin\frac{n\pi}{\ell}x$$

(c) A string is stretched and fastened to two points ℓ apart. Motion is started by displacing the string in the form $y = h(\ell x + x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t.

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