Subject Code: KAS303

PAPER ID-411527

Roll No:

BTECH
(SEM III) THEORY EXAMINATION 2021-22
MATHEMATICS-III

Time: 3 Hours *Total Marks: 100*

Notes:
- *Attempt all Sections and Assume any missing data.*
- *Appropriate marks areallotted to each question, answer accordingly.*

SECTION-A	Attempt **All** of the following Questions in brief Marks**(10X2=20)**	**CO**
Q1(a)	Find the Laplace transform of $\int_0^t e^{-t} \, cost \, dt$.	1
Q1(b)	Evaluate: $L^{-1}(\frac{e^{-2p}}{p^2})$.	1
Q1(c)	State Modulation theorem.	2
Q1(d)	Find the Z-transform of $f(k) = \frac{1}{k}, \ \ k \geq 1$.	2
Q1(e)	Prove that the statement $(p \rightarrow q) \rightarrow (p \wedge q)$ is a contingency.	3
Q1(f)	Show that every sub- group H of an abelian group G is normal.	3
Q1(g)	How many bit strings of length 8 either start with 1 or end with two bits 00?	4
Q1(h)	Given $f(x) = 3x - 2$, find $f^{-1}(x)$.	4
Q1(i)	Define Maximal element and Minimal element of Poset.	5
Q1(j)	Prove the Boundedness (NULL) law i.e. $a * 0 = 0$.	5

SECTION-B	Attempt **ANY THREE**of the following Questions Marks**(3X10=30)**	**CO**
Q2(a)	Solve the following simultaneous equations by Laplace transform $3\frac{dx}{dt} - y = 2t$, $\frac{dx}{dt} + \frac{dy}{dt} - y = 0$ with the conditions $x(0) = 0, y(0) = 0$.	1
Q2(b)	i) Find the Fourier transform of $F(x) = \begin{cases} 1, & \lvert x \rvert < a \\ 0, & \lvert x \rvert > a \end{cases}$. Hence evaluate $\int_0^\infty \frac{sinp}{p} \, dp$. ii) Using Parseval's identity, show that $\int_0^\infty \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx = \frac{\pi}{2(a+b)}$. Hence find $\int_0^\infty \frac{x^2}{(x^2+1)^2} dx$.	2
Q2(c)	Consider a ring $(R, +, *)$ defined by $a * a = a$. Determine whether the ring is commutative or not.	3
Q2(d)	Solve the difference equation $y_K - y_{K-1} - 6y_{K-2} = -30$. Given that $y_0 = 20, \ y_1 = -5$.	4
Q2(e)	In a Boolen algebra B if $b + a = c + a$ and $b + a' = c + a'$ then $b = c$. Also if $ba = ca$ and $ba' = ca'$ then $b = c$.	5

SECTION-C	Attempt **ANY ONE** following Question Marks **(1X10=10)**	**CO**
Q3(a)	Draw the graph and find the Laplace transform of the triangular wave function of period of 2c given by $f(t) = \begin{cases} t, & 0 < t \leq c \\ 2c - t & c < t < 2c \end{cases}$.	1
Q3(b)	Prove that: $L^{-1}\left\{\frac{1}{(p^2+1)^3}\right\} = \frac{1}{8}[(3 - t^2)sint - 3t \, cost]$	1

SECTION-C	Attempt **ANY ONE** following Question Marks **(1X10=10)**	**CO**
Q4(a)	Determine the distribution of temperature in the semi- infinite medium $x \geq 0$ when the end $x = 0$ is maintained at zero temperature and the initial distribution of temperature is F(x).	2
Q4(b)	Solve by Z-transform the difference equation $y_{k+2} + 6y_{k+1} + 9y_k = 2^k$; $y_0 = y_1 = 0$.	2

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SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q5(a)	Consider the following argument and determine whether it is valid. Either I will get good marks or I will not graduate. If I did not graduate I will go to Canada. I get good marks. Thus, I would not go to Canada.		3
Q5(b)	Translate the following into symbolic form and test the validity of the argument. If 6 is even then 2 does not divide 7. Either 5 is not prime or 2 divides 7. But 5 is prime, therefore ,6 is odd.		3

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q6(a)	Among the first 1000 positive integers: (a) Determine the integers which are not divisible by 5, nor by 7, nor by 9. (b) Determine the integers divisible by 5, but not by 7, not by 9.		4
Q6(b)	Prove by Mathematical Induction $\frac{1}{1\,.\,3}+\frac{1}{3\,.\,5}+\frac{1}{5\,.7}+\cdots\ldots..+\frac{1}{(2n-1).\,(2n+1)}=\frac{n}{2n+1}$.		4

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q7(a)	If L be any Lattice, then for any $a, b, c\ \in L$. Prove the following: (i) $a \vee a = a$. (ii) $a \vee b = b \vee a$(iii) $a \vee (b \vee c) = (a \vee b) \vee c$ (iv) $a \vee (a \wedge b) = a$.		5
Q7(b)	For two posets A and B: (i) Show that if $f: A \to B$ is an order preserving, then f is one to one. (ii) Show that if $f: A \to B$ is an order isomorphism, then there is an order isomorphism $g : B \to A$.		5