(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1226 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2013-14

MATHEMATICS – III

[Branches : ME, AE, MT, TT, TE, TC, FT, CE CH]

Time : 3 Hours *Total Marks : 100*

Note :– Attempt **all** questions from each Section as indicated. The symbols have their usual meaning.

SECTION–A

1. Attempt **all** parts of this Section. Each part carries 2 marks : **(2×10=20)**

(a) Define Conformal Transformation.

(b) Find residue of $f(z)=\frac{z^2}{z^2+3z+2}$ at the pole -1.

(c) Define Fourier Transform of a function $f(x)$.

(d) Find the Z-Transform of $\{a^k\}$, $k \geq 0$.

(e) Define coefficients of Skewness.

(f) What is Total Probability Theorem ?

(g) Define Spline Function.

(h) Show that $\delta = E^{\frac{1}{2}} + E^{-\frac{1}{2}}$.

(i) Define rate of convergence.

(j) What do you mean by initial value problem ?

SECTION–B

2. Attempt any **three** parts of this Section. **(10×3=30)**

(a) State and prove Cauchy integral formula. Also evaluate

$$\oint_c \frac{z^2+1}{z^2-1}\,dz \text{, where c is the circle :}$$

(i) $|z-1|=1$

(ii) $|z|=\frac{1}{2}$.

(b) Using Fourier Transform, solve $\frac{\partial y}{\partial t}=\frac{\partial^2 y}{\partial x^2}, -\infty < x < \infty,$ $t < 0;\ y(x, 0) = f(x)$.

(c) The first four moments of a distribution about the value '4' of the variable are –1.5, 17, –30 and 108. Find the moments about mean and about origin. Also find Skewness and Kurtosis.

(d) Use Gauss-Seidel method to solve the following system of simultaneous equations :

$$9x + 4y + z = -17$$

$$x - 2y - 6z = 14$$

$$x + 6y = 4$$

Perform four iterations.

(e) Given $\frac{dy}{dx}=y-x$, $y(0) = 2$. Find $y(0.1)$ and $y(0.2)$ correct to four decimal places by Runge-Kutta fourth method.

SECTION–C

Note :– All questions of this Section are compulsory. Attempt any **two** parts from each question : **(5×2×5=50)**

3. (a) Verify Cauchy's theorem by integrating z^3 along the boundary of a square with vertices at $1+i$, $1-i$, $-1+i$ and $-1-i$.

(b) Evaluate the following integral by using complex integration.

$$\int_0^{\pi} \frac{\cos 2\theta}{5+4\cos\theta}\, d\theta .$$

(c) Determine the analytic function $f(z) = u + iv$, in terms of z, whose real part is $e^{-x}(x \sin y - y \sin y)$.

4. (a) Find the Fourier transform of :

$$F(x)=\begin{cases}1, |x|<a\\ 0, |x|>a\end{cases}. \text{ Hence evaluate}$$

(i) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin ap \cos px}{p}\, dp$

(ii) $\displaystyle\int_0^{\infty} \frac{\sin p}{p}\, dp$.

(b) Find the inverse Z-transform of :

$$F(z) = \frac{1}{(z-3)(z-2)} \text{ for}$$

(i) $|z| < 2$

(ii) $2 < |z| < 3$

(iii) $|z| > 3$.

(c) Solve by Z-transform the difference equation :

$y_{k+2} + 6y_{k+1} + 9y_k = 2^k$; $(y_0 = y_1 = 0)$.

5. (a) State and prove Baye's theorem.

(b) A continuous random variables X has a p.d.f. $f(x) = 3x^2$, $0 \le x \le 1$. Find a and b such that :

(i) $P(X \le a) = P(X > a)$, and

(ii) $P(X > b) = .05$.

(c) Fit a Poisson distribution to the following data and calculate theoretical frequencies :

Death	0	1	2	3	4
Frequencies	122	260	15	2	1

6. (a) Find a positive value of $(17)^{1/3}$ correct to four decimal places by Newton-Raphson method.

(b) Obtain cubic spline for the following data :

x	0	1	2	3
f (x)	1	2	33	244

With the end conditions $M_0 = M_3 = 0$ for (0, 1). Hence compute $f(0.5)$.

(c) From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46.

Age	45	50	55	60	65
Premium (in rupees)	114.84	96.16	83.32	74.48	68.48

7. (a) The table given below reveals the velocity 'v' of a body during the time 't' specified. Find its acceleration at t = 1.1.

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

(b) Evaluate $\int_0^6 \frac{e^x}{x+1}\,dx$ by Simpson's 3/8th rule.

(c) Using Milne's method, solve $\frac{dy}{dx} = 1 + y^2$ with initial condition y(0) = 0, y(0.2) = 0.2027, y(0.4) = 0.4228, y(0.6) = 0.6841, obtain y(0.8).