**Printed pages: 3** **Roll No.** | | | | | | | | | | | **Sub Code: NAS-301**
**Paper ID: 9 0 6 3**

**B-TECH**
**(SEM. III) THEORY EXAMINATION 2017-18**
**Engg. Mathematics-III**

**Time: 3 Hours** **[Total Marks: 100]**

**Note: 1.** Attempt all Sections. If require any missing data; then choose suitably.

## SECTION-A

**1. Attempt *all* questions in brief.** **2x10 = 20**

a) Find inverse Z-transformation of $\frac{z}{z^2-1}$.

b) If $u(x,y) = x^2 - y^2$ , prove that u satisfies Laplace equation.

c) Evaluate $\int_C \frac{z^2+1}{z^2-1} dz$ *where C is circle* $|z| = 3/2$ .

d) Expand $\frac{1}{(z+1)(z+3)}$ in the regions $|z| < 1$.

e) Estimate the production for 1964 and 1966 from the following data

| Year: | 1961 | 1962 | 1963 | 1964 | 1965 | 1966 | 1967 |
|---|---|---|---|---|---|---|---|
| Production: | 200 | 220 | 260 | --- | 350 | --- | 430 |

f) State Gregory-Newton divided difference interpolation formula.

g) Find Z-transformation of $f(k) = \begin{cases} 1 , & k = 0 \\ 0, & k \neq 0 \end{cases}$

h) State Cauchy's integral theorem.

i) Prove that: $\Delta \log f(x) = \log[1 + \frac{\Delta f(x)}{f(x)}]$

j) Define kurtosis of a distribution.

## SECTION-B

**2. Attempt any three parts of the following:** **$(3 \times 10 = 30)$**

a) Find the Fourier transform of $F(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ ,

hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$

b) Examine the nature of the function $f(z) = \begin{cases} \frac{x^2 y^5 (x+iy)}{x^4+y^{10}}; z \neq 0 \\ 0 \qquad\qquad z = 0 \end{cases}$

In the region including the origin.

c) Solve the following system of linear equations by Crout's Method :

x + y + z = 3; 2x - y + 3z = 16; 3x + y - z = -3

d) Find the rank correlation coefficient of marks of A and B from the following data:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Marks A | 15 | 20 | 27 | 13 | 45 | 60 | 20 | 75 |
| Marks B | 50 | 30 | 55 | 30 | 25 | 10 | 30 | 70 |

e) Solve the following differential equations using Runge- Kutta method :

Solve $\frac{dy}{dx} = \frac{1}{x+y}$ for x = 0.5, to x = 1, h = 0.5 with y(0) = 1.

## SECTION-C

**3. Attempt any two parts of the following:** **(2 × 5 = 10)**

(a) Using Lagrange's interpolation formula, find y(10) from the following table:

| | | | | |
|---|---|---|---|---|
| x : | 5 | 6 | 9 | 11 |
| y : | 12 | 13 | 14 | 16 |

(b) The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate $\beta_1$ *and* $\beta_2$ and comment upon the skewness and kurtosis of the distribution.

(c) Using the Fourier integral transformation, show that

$$e^{-ax} = \frac{2a}{\pi}\int_0^\infty \frac{\cos sx}{s^2+a^2}\,ds, \quad a>0,\ x \geq 0.$$

**4. Attempt any two parts of the following:** **(2 × 5 = 10)**

(a) Evaluate by Cauchy integral formula $\oint_C \frac{z^2-2z}{(z+1)^2(z^2+4)}\,dz$ whereC is the circle $|z|=3$.

(b) Solve $x^3 - 5x + 3 = 0$ by using Regula – Falsi method.

(c) Using the Z-transform solve the following difference equations:

$y_{k+2} + 6y_{k+1} + 9y_k = 2^k$ *given* $y_{(0)} = 0,\ y_{(1)} = 0$.

**5. Attempt any two parts of the following:** **(2 × 5 = 10)**

(a) If $f(z) = u + iv$ is analytic function and $u - v = e^x(\cos y - \sin y)$, find f (z) in terms of z.

(b) Using poisson distribution, find the probability that the ace of spades will be drawn from a pack of well shuffled cards at least once in 104 consecutive trails.

(c) Find $\int_0^6 \frac{e^x}{1+x}dx$ approximately using Simpson's 3/8 rule on integration.

**6. Attempt any two parts of the following:** **$(2 \times 5 = 10)$**

(a) The table given below reveals the velocity 'v' of a body during the time't' specified. Find its acceleration at t=1.1 .

| t | : | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 |
|---|---|---|---|---|---|---|
| v | : | 43.1 | 47.7 | 52.1 | 56.4 | 60.8 |

(b) Using Complex integration method to evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$.

(c ) Compute $f'(3)\ from$ the following table

| x | : | 1 | 2 | 4 | 8 | 10 |
|---|---|---|---|---|---|---|
| y | : | 0 | 1 | 5 | 21 | 27 |

**7. Attempt any two parts of the following:** **$(2 \times 5 = 10)$**

(a) Using picards method obtain y for x=0.2, Given $\frac{dy}{dx} = x - y$ with initial condition y=1, when x=0.

(b) Discuss the Newton-Raphson method and prove that the order of convergence of Newten-Raphson method is quadratic.

(c) Fit a relation $y = ax + \frac{b}{x}$ which satisfies the following data , using method of least square.

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| y | 5.4 | 6.2 | 8.2 | 10.3 | 12.6 | 14.8 | 17.2 | 19.5 |