Printed Pages : 5

NAG201

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 180219

Roll No.

B. Tech.

(SEM. II) THEORY EXAMINATION, 2014-15

ENGINEERING MATHEMATICS - II

Time : **3** Hours] [Total Marks : **100**

Note : Attempt all questions.

SECTION A

1 Attempt all parts of this question : **10×2=20**

(a) Find Curl of $\overline{f} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$ at the point $(1, 0, 2)$.

(b) State Green's theorem.

(c) Define harmonic function.

(d) Find the period of $\sin nx$.

(e) Find a_0 if $f(x) = x$ is expanded in half range Fourier cosine series.

(f) Form the partial differential equation from $z = ax + by + a^2 + b^2$.

(g) Write the one dimensional heat equation and its solution.

(h) Solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ by method of separation of variables.

(i) Define skewness.

(j) Find the median of the following item :
3, 2, 5, 9, 7, 4, 3, 8.

SECTION B

2 Attempt any three parts of the following : **3×10=30**

(a) Using Green's theorem evaluate :

$$\int_C \left[\left(x^3 - xy^3\right)dx + \left(y^2 - 2xy\right)dy\right]$$ where C is the square having vertices at the points $(0, 0), (2, 0), (2, 2)$ and $(0, 2)$.

(b) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also express $f(z)$ in terms of z.

(c) Find the Fourier series to represent the function

$$f(x) = \begin{cases} -k & , \text{ when } -\pi < x < 0 \\ k & , \text{ when } 0 < x < \pi \end{cases}$$

hence show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$

(d) Find the temperature distribution in a rod of length a which is totally insulated including the ends if the initial temperature is $x(a-x)$.

(e) Using the method of least squares, fit a straight line to the following data :

x	0	1	2	3	4
y	1.0	2.9	4.8	6.7	8.6

SECTION C

Note : Attempt any two parts from each questions. **(2×5)×5=50**
All questions are compulsory.

3 (a) Find grad ϕ when ϕ is given by $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, -1)$.

(b) Prove that $\vec{W} = \frac{1}{2} curl\, \vec{V}$, where $\vec{W}$ is angular velocity and $\vec{V}$ is linear velocity.

(c) Find $\iint_S \vec{F} \cdot \hat{n}\, dS$, where

$\vec{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$ and S is the surface of the sphere having centre at $(3, -1, 2)$ and radius 3.

4 (a) Evaluate $\lim_{z \to 2e^{\pi/3}} \left[\frac{z^3 + 8}{z^4 + 4z^2 + 16} \right]$

(b) Discuss the analyticity of $f(z) = z\,\bar{z}$

(c) Find the analytic function $f(z)$ in terms of z whose imaginary part is $\cos x \cosh y$.

5 (a) Find the half range cosine series for $f(x) = x$ in $0 < x < c$.

(b) Solve : $\left(D^2 + 3DD' + 2D'\right) z = \cos(x + 3y)$

(c) Solve : $\left(D^2 + 2D' - 3\right) z = e^{2x-3y}$

6 (a) Solve by the method of separation of variables

$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$$

(b) Solve : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, in a rectangle with $u(0, y) = 0$, $u(a, y) = 0$, $u(x, b) = 0$ and $u(x, 0) = f(x)$.

(c) Find the temperature distribution in a rod of length ℓ whose ends are kept at zero temperature and the initial temperature is u_0.

7 (a) Calculate the mean deviation from the following data :

Class interval :	0-10	10-20	20-30	30-40	40-50	50-60
Frequency :	3	5	11	5	4	9

(b) Calculate the coefficient of skewness from the following data :

x :	1	2	3	4	5
f :	8	28	56	70	56

(c) From the following data calculate median :

x :	15	25	35	45	55	65
f :	18	29	46	62	96	56