Paper Id: **180117**   **Roll No:**

B. TECH.
(SEM-I) THEORY EXAMINATION 2019-20
ENGINEERING MATHEMATICS- I

Time: 3 Hours   *Total Marks: 100*

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1.   **Attempt *all* questions in brief.**   **10 x 2 = 20**

     a)   Find the derivative of $y = x^3 - 2x^2 + 4$ with respect to x.

     b)   Examine the continuity of $f(x) = x^2 - x + 1$ at $x = 1$.

     c)   If $u = x^2 - y^2$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

     d)   If $x = r\cos\theta$ and $y = r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

     e)   Define Gamma function.

     f)   Evaluate $\iint e^{x-y}\, dx\, dy$ over the triangle bounded by x=0,y=0 and x+y = 1.

     g)   Find order and degree of differential equation $(\frac{dy}{dx})^2 + 5y = x$.

     h)   Find the I.F. of $\frac{dy}{dx} + xy = e^{2x}$.

     i)   Find the Rank of the matrix A=$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix}$.

     j)   The Eigen value of matrix A are 1,-2,3 then find the Eigen value of A^3.

SECTION B

2.   **Attempt any *three* parts of the following:**   **10 x 3 = 30**

     a)   Differentiate $(x)^x$ with respect to x.

     b)   Expand $f(x) = e^{\sin x}$ in powers of $(x - \pi/2)$ by Taylor's theorem

     c)   Solve $\dfrac{d^2 y}{dx^2} + 5\dfrac{dy}{dx} + 6y + \sin 2x + \cos 2x$.

     d)   Evaluate the area enclosed between the parabola $y = x^2$ and the straight line $y = x$.

     e)   Find the characteristic equation of the matrix and verify Cayley Hamilton theorem. Hence find A^{-1}.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

SECTION C

3.   **Attempt any *two* parts of the following:**   **5 x 2 = 10**

     a)   Differentiate log (ax+b) with respect to x.

     b)   Evaluate $\lim\limits_{x \to 1} \frac{x^3 - 1}{x - 1}$.

     c)   Evaluate $\int \frac{2x+5}{x^2+5x+9}\, dx$.

4.   **Attempt any *two* parts of the following:**   **5 x 2 = 10**

     a)   If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, Show that $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})u = 3/(x+y+z)$.

     b)   What is the degree of homogeneous function $u(x,y) = x^2(x^2-y^2)^{1/3} / (x^2+y^2)^{2/3}$

     c)   If $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$, find $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$.

5.   **Attempt any *two* parts of the following:**   **5 x 2 = 10**

   a) Evaluate $\iint x\, y\, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

   b) Change the order of integration $\int_0^2 \int_0^x dy\, dx$.

   c) Prove that $\beta(m, n) = \left(\frac{\Gamma m\ \Gamma n}{\Gamma(m+n)}\right)$.

6.   **Attempt any *two* parts of the following:**   **5 x 2 = 10**

   a) Solve $\frac{dy}{dx} = x \tan y$

   b) Solve $(D^2 - 4D + 4)\, y = x^3\, e^{2x}$.

   c) Using method of variation of parameter to solve $y'' + y = \sec x$.

7.   **Attempt any *two* parts of the following:**   **5 x 2 = 10**

   a) Find inverse of the matrix by using elementary transformation $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

   b) Find the Eigen values of the following matrix $A = = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

   c) Find the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 5 \\ 4 & -9 & 10 \\ 3 & -6 & 15 \end{bmatrix}$