

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9929

Roll No.

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**B. Tech.**

(SEM. II) THEORY EXAMINATION 2010-11

**MATHEMATICS-II**

Time : 3 Hours

Total Marks : 100

**Note :** Attempt all questions. All questions carry equal marks.1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Solve the equation  $x \frac{dy}{dx} - \frac{1}{2}y = x + 1$  and prove that the only solution for which  $x$  and  $y$  can attain the value unity is given by  $y = 2x + \sqrt{x} - 2$ .

(b) Solve  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = \sin 3x + \cos 2x$ .

(c) Solve  $(x+1)^2 \frac{d^2y}{dx^2} + (x+1) \frac{dy}{dx} = (2x+3)(2x+4)$ .

(d) Solve the simultaneous equations :

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t$$

given  $x(0) = 1, y(0) = 0$ .

- (e) Solve the following differential equation by reducing into normal form

$$\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + (x^2 + 2)y = e^{\frac{1}{2}(x^2+2x)}$$

- (f) An RL circuit has an e.m.f. given (in volts) by  $4 \sin t$ , a resistance of 100 ohms, an inductance of 4 henry and no initial current. Find the current at any time  $t$ .

2. Attempt any **two** parts of the following : (10×2=20)

- (a) Using Frobenius method, obtain a series solution about  $x = 0$  for the differential equation :

$$2x^2y'' - xy' + (1 - x^2)y = 0.$$

- (b) (i) Prove that  $\int_{-1}^1 P_m(x) P_n(x) dx = \frac{2}{2n+1}$ ,  $m = n$ .

- (ii) Prove that  $n P_n(x) = x P_n'(x) - P_{n-1}'(x)$ .

- (c) (i) Prove that  $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[ \left( \frac{3-x^2}{x^2} \right) \sin x - \frac{3 \cos x}{x} \right]$

- (ii) Prove that  $J_n''(x) = \frac{1}{4} [J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)]$ .

3. Attempt any **four** parts of the following : (5×4=20)

- (a) State first shifting property for Laplace transform. Hence find the Laplace Transform of

$$f(t) = t^2 e^t \sin 4t.$$

(b) Use Laplace transform to evaluate :

$$\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$$

(c) Express the following functions in terms of unit step function and hence find its Laplace transform :

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ \sin 2t, & \pi < t \end{cases}$$

(d) Find the function whose Laplace transform is :

$$F(s) = \frac{s-1}{s^2(s-7)}$$

(e) Solve the following integral equation :

$$y(t) = 1 - \sinh t + \int_0^t (1+u) y(t-u) du$$

(f) Using Laplace transform, solve :

$$\frac{d^2x}{dt^2} + 9x = \sin 2t, \quad x(0) = 1, \quad x'(0) = 0$$

4. Attempt any two parts of the following : (10×2=20)

(a) Find the Fourier series of the periodic function  $f(x)$  with period  $2\pi$  defined as follows :

$$f(x) = \begin{cases} 0, & -\pi < x \leq 0 \\ x, & 0 \leq x \leq \pi \end{cases}$$

Hence prove that  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

(b) Given the function  $f(x) = x$ ,  $0 < x < 1$ , find :

(i) Fourier cosine series for  $f(x)$ ,

(ii) Fourier sine series for  $f(x)$ .

(c) Solve  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = xy + e^{x+2y}$

5. Attempt any **two** parts of the following : (10×2=20)

(a) Use the method of separation of variables to obtain the general solution of

$$c^2 \frac{\partial^2 z}{\partial x^2} = \frac{\partial u}{\partial t}$$

that tends to zero as  $t \rightarrow \infty$  for all  $x$ .

(b) Find the steady state temperature distribution in a semicircular plate of radius  $a$ , insulated on both faces with its curved boundary kept at a constant temperature  $U_0$  and its bounding diameter is kept at zero temperature.

(c) A string is stretched and fastened to two points distance  $L$  apart. Find the displacement of the string at any point  $x$  and at any time  $t$  if the motion is started by displacing the string in the form  $y = a \sin^3 \frac{\pi x}{L}$  from which it is released at time  $t = 0$ .