(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9929 Roll No.

B. Tech.

(SEM. II) EXAMINATION, 2007-08

MATHEMATICS - II

Time : 3 Hours] [Total Marks : 100

Note : (1) *Attempt **all** questions.*

(2) *All questions carry **equal** marks.*

(3) *In case of numerical problems assume data wherever not provided.*

(4) *Be precise in your answer.*

1 Attempt any **four** parts of the following : 5×4=20

(a) Solve $\left(y+\sqrt{x^2+y^2}\right)dx - x\,dy = 0$,

$y(1)=0$.

(b) Solve the following differential equation :

$$(\cos x - x\cos y)dy - (\sin y + y\sin x)dx = 0$$

(c) Find the solution of following differential equation :

$$(D^2-4D-5)y = e^{2x} + 3\cos(4x+3)$$

where $D = \frac{d}{dx}$.

(d) Solve the following simultaneous differential equations :

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0$$

Also show that $x = y = 0$ when $t = 0$.

(e) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\log x.$$

(f) An inductance (L) of **2.0 H** and a resistance (R) of $20\ \Omega$ are connected in series with an e.m.f. E volt. If the current (i) is zero, when $t=0$, find the current (i) at the end of **0.01** second if $E = 100\ V$, using the following differential equation :

$$L\frac{di}{dt} + iR = E.$$

2 Attempt any **four** parts of the following : **5×4=20**

(a) Using Laplace transform, evaluate

$$\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt.$$

(b) State second shifting theorem for Laplace transform and hence find the Laplace transform of the following function

$$F(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$$

(c) Using convolution theorem, find the inverse Laplace transform of the following $\frac{s}{(s^2+a^2)^3}$.

(d) Solve the following simultaneous differential equations by Laplace transform

$$\frac{dx}{dt} + 4\frac{dy}{dt} - y = 0$$

$$\frac{dx}{dt} + 2y = e^{-t}$$

with condition $x(0) = y(0) = 0$.

(e) Solve the following differential equation using Laplace transform $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x} \sin x$

where $y(0) = 0, \left(\frac{dy}{dx}\right)_{x=0} = 1$.

(f) Using unit step function, find the Laplace transform of :

(i) $(t-1)^2 \cdot u(t-1)$

(ii) $\sin t \cdot u(t-\pi)$.

3 Attempt any **two** parts of the following : **10×2=20**

(a) Solve the following differential equation in series :

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0.$$

(b) Show that

(1) $\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{n+1}$

(2) $P_{2n}(0) = \frac{(-1)^n (2n)!}{2^{2n} (n!)^2}$

where $P_n(x)$ is the Legendre polynomial of degree n.

(c) Show that

(1) $J_2^1(x) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$

(2) $J_0^2 + 2\left(J_1^2 + J_2^2 + J_3^2 + ...\right) = 1$

where $J_n(x)$ is the Bessel's polynomial of degree n and dash denote the differentiation.

4 Attempt any **two** parts of the following : **10×2=20**

(a) Obtain a Fourier series to represent $f(x) = x^2$ in the interval $(0, 2\pi)$ and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}.$$

(b) Examine whether the function $f(x) = x \sin x$ is even or odd. Hence expand it in the form of Fourier series in the interval $(-\pi, \pi)$.

(c) Solve the following partial differential equations :

(1) $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$

(2) $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \cos 2y$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

5 Attempt any **two** parts of the following : **10×2**

(a) Solve the following equation by the method of separation of variables :

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y} \text{ where } u(0, y) = 8e^{-3y}.$$

(b) Solve the following Laplace equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in a rectangle with $u(0, y) = 0$, $u(a, y) = 0$, $u(x, b) = 0$ and $u(x, 0) = f(x)$ along x–axis.

(c) Assuming the resistance of wire (R) and conductance to ground (C_T) are negligible, find the voltage $v(x, t)$ and current $i(x, t)$ in a transmission line of length l, t seconds after the ends are suddenly grounded. The initial conditions are $v(x, 0) = v_0 \sin\left(\frac{\pi x}{l}\right)$ and $i(x, 0) = i_0$.
