Printed Pages : 4 **TAS – 204**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9929 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. II) EXAMINATION, 2006-07

MATHEMATICS - II

(SPECIAL CARRYOVER EXAMINATION)

Time : 3 Hours] *[Total Marks :* ***100***

Notes : *(i) Attempt* ***all*** *questions.*
(ii) All questions carry ***equal*** *marks.*

1 Attempt any **four** parts of the following :

(a) Solve the first order differential equation

$$\frac{dy}{dx}+\frac{y}{\left(1-x^2\right)^{3/2}}=\frac{x+\sqrt{1-x^2}}{\left(1-x^2\right)^2}.$$

(b) Integrate $\left(1+x^2\right)\frac{dy}{dx}+2xy-4x^2=0$.
Obtain equation of the curve satisfying this equation and passing through the origin.

(c) Find the complete solution of the differential equation $\left(D^2-1\right)y=xe^x+\cos^2 x$.

(d) Solve the following simultaneous differential equation $\frac{dx}{dt}=-wy,\ \frac{dy}{dt}=wx$.
Also show that the point (xy) lies on a circle.

(e) If $\frac{d^2x}{dt^2}+\frac{g}{b}(x-a)=0$ (a, b and g being positive constants) and $x=a'$ and $\frac{dx}{dt}=0$ when $t=0$. Show that

$$x=a+(a'-a)\cos\left(\sqrt{\frac{g}{b}}t\right).$$

(f) A particle begins to move from a distance $'a'$ towards a fixed centre, which repels it with retardation (μx). If its initial velocity is $a\sqrt{\mu}$, show that it will continually approach the fixed centre, but will never reach it.

2 Attempt any **two** parts of following :

(a) Prove the orthogonal property of Legendre polynomial

$$\int_{-1}^{1} P_m(x)P_n(x)\,dx=\frac{2}{(2n+1)}\delta_{mn}$$

where Kronecker delta δ_{mn} is

$$\delta_{mn}=\begin{cases}0 & \text{if } m\neq n\\ 1 & \text{if } m=n\end{cases}$$

(b) Find the series solution of Bessel's differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$

(c) Prove the recurrent relations :

(i) $\frac{d}{dx}\left[x^{-n}J_n(x)\right]=-xJ_{n+1}(x)$

(ii) $\frac{2h}{x}J_n(x)=J_{n-1}(x)+J_{n+1}(x)$

3 Attempt any **four** parts of the following :

(a) Find the inverse Laplace transforms of following functions :

(i) $$\frac{(14s+10)}{(49s^2+28s+13)}$$

(ii) $$\frac{1}{(s^4+4)}$$

(b) Find the Laplace transform of "Saw-tooth wave" function $f(t)$ which is periodic with period 1 and defined as $f(t) = kt$ in $0 < t < 1$.

(c) Find the Laplace transform periodic function

$$f(t) = \begin{cases} t & 0 < t < a \\ -t+2a & a < t < 2a \end{cases}$$

(d) Using convolution theorem, find the inverse of the function $\dfrac{1}{(s^2+a^2)^2}$.

(e) Using Laplace transform, evaluate the following integrals

(i) $$\int_0^\infty \frac{e^{-t}\sin\sqrt{3}t}{t}\,dt$$

(ii) $$\int_0^\infty \left(\frac{e^{-2t}-e^{-4t}}{t}\right)dt$$

(f) Using Laplace transform, solve the equation

$$L\frac{dI}{dt} + RI = Ee^{-at}, I(0) = 0$$

where L, R, E and a are constants.

4 Attempt any **two** parts of following :

(a) Obtain the fourier series expansion of

$$f(x) = \left(\frac{\pi - x}{2}\right) \text{ for } (0 < x < 2).$$

(b) If $f(x) = \sin\left(\frac{\pi x}{L}\right)$ in $(0 < x < L)$.

Find the fourier cosine series. Graph the corresponding periodic continuation of $f(x)$.

(c) Solve the partial deferential equation by method of separation of variables

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0.$$

5 Attempt any **two** parts of following :

(a) Find the temperature in a bar of length 2 whose each are kept at zero and lateral surface insulated of the intial temperature is

$$\left[\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right)\right].$$

(b) Find the steady state temperature distribution in a rectangular them plate with its two surfaces insulated and with the condution.

$$u(0, y) = 0, u(x, 0) = 0, u(a, y) = g(y)$$

$$u(x\, b) = f(x)$$

(c) Solve $\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}$ assuming that the intial voltage is $V_0 \sin\left(\frac{\pi x}{l}\right)$ $V_1(x_0) = 0$ and $V = 0$ at the ends, $x = 0$ and $x = l$ for all t.