(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9929
9919

Roll No. | | | | | | | | | | |

B.Tech.

SECOND SEMESTER EXAMINATION, 2005-2006

MATHEMATICS - II

Time : 3 Hours *Total Marks : 100*

Note : *(i)* *Attempt **ALL** questions.*

(ii) *All questions carry equal marks.*

(iii) *In case of numerical problems assume data wherever not provided.*

(iv) *Be precise in your answer.*

1. Attempt *any four* parts of the following : **(5x4=20)**

(a) Solve the following differential equation :

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$$

(b) Solve : $x^2dy + y\,(x+y)\,dx = 0$

(c) Solve the following simultaneous differential equations

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

(d) Solve the following differential equation by changing the independent variable

$$x\frac{d^2y}{dx^2} + (4x^2 - 1)\frac{dy}{dx} + 4x^3y = 2x^3$$

(e) Solve the following differential equation by method of variation of parameters

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$$

(f) The equations of electromotive force in terms of current i for an electrical circuit having resistance R and a condenser of capacity C, in series, is

$$E = Ri + \int \frac{i}{C}\,dt$$

Find the current i at any time t, when $E = E_o \sin\omega t$.

2. Attempt *any four* parts of the following : **(5x4=20)**

(a) If $L\{f(t)\} = \bar{f}(s)$ and

$$g(t) = \begin{cases} f(t-a), & t > a \\ 0, & 0\,t < a \end{cases}$$

then prove that $L\{g(t)\} = e^{-as}\bar{f}(s)$

(b) If $L\{\cos^2 t\} = \dfrac{S^2+2}{S(S^2+4)}$

find $L\{\cos^2 at\}$

(c) State convolution theorem and hence evaluate

$$L^{-1}\left\{\frac{S^2}{(S^2+a^2)(S^2+b^2)}\right\}$$

(d) Using Laplace transform, find the solution of the initial value problem.

$$\frac{d^2y}{dt^2}+9y=6\cos 3t$$

Where $y(0)=2,\ y'(0)=0$

(e) Solve the simultaneous differential equations by Laplace transform

$Dx-y=e^t$

$Dy+x=\sin t$

(f) A function f(t) obeys the equation

$$f(t)+2\int_0^t f(t)\,dt=\cosh 2t$$

Find the Laplace transform of f (t).

3. Attempt *any two* parts of the following : **(10x2=20)**

(a) Find the Fourier series of the function defined as

$$f(x)=\begin{cases} x+\pi, & \text{for } 0\le x\le\pi \\ -x-\pi & \text{for } -\pi\le x<0 \end{cases}$$

and f $(x+2\pi)$ = f (x).

(b) Find the Fourier half-range cosine series of the function

$$f(t)=\begin{cases} 2t, & 0<t<1 \\ 2(2-t), & 1<t<2 \end{cases}$$

(c) (i) Solve $(p^2+q^2)y=qz$ where p and q are usual notations of PDE.

(ii) Solve the linear partial differential equation

$$\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x^2 y}+\frac{\partial^2 z}{\partial y^2}=\sin(2x+3y)$$

OR

(i) Find the Fourier transform of

$$f(x)=\begin{cases} 1, & |x|<a \\ 0, & |x|>a \end{cases}$$

(ii) Find z – transform of sin αk where $k\geq 0$.

Note : ***Following Question. Number 4 and 5 are for New Syllabus only (TAS - 204/MA - 202 (New).***

4. Attempt *any two* parts of the following : **(10x2=20)**

(a) Solve the following differential equation in series.

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$$

(b) Show that

(i) $nPn = xP_n' - P_{n-1}'$

(ii) $\int_{-1}^{+1}(1-x^2)P_m' P_n'\,dx=0, m\neq n$

where dashes denote differentiation *w.r.t. x*.

(c) Prove that

(i) $xJ_n'(x) = -nJ_n(x)+xJ_{n-1}(x)$

(ii) $x^2 J_n''(x)=(n^2-n-x^2)J_n(x)+xJ_{n+1}(x), n=0, 1, 2, ...$

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Using the method of separation of variable, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x, 0) = 6\, e^{-3x}$

(b) Determine the solution of one dimensional heat equation

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

Subject to the boundary conditions $u(0, t) = 0$, $u(l, t) = 0$ $(t>0)$ and the initial condition $u(x, 0) = x$, l being the length of the bar.

(c) If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity

$\left(\frac{dy}{dt}\right)_{t=0} = b \sin^3 \frac{\pi x}{l}$, find the displacement $y(x, t)$.

Note : ***Following Question. Number 4 and 5 are for Old Syllabus only (MA - 202 (Old).***

4. Attempt *any two* parts of the following : **(10x2=20)**

(a) Evaluate the integral

$$\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\, dy\, dx$$

(b) Evaluate $\iiint xyz \, dz dy dx$ over the volume enclosed by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$

(c) Apply Dirichlet's integral to find the mass of an octant of the ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, the density at any point being $\rho = kxyz$.

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Solve the following by Cardon's method

$x^3 - 3x^2 + 12x + 16 = 0$

(b) By the method of least squares, find the curve $y = ax + bx^2$ that best fits the following data

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

(c) Solve $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ by Ferrari's method.

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