(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610

Roll No. | | | | | | | | | | |

B.Tech.

(SEMESTER-II) THEORY EXAMINATION, 2011-12

MATHEMATICS – II

Time : 3 Hours] ***[Total Marks : 100***

Note : Attempt questions from each Section as indicated. The symbols have their usual meaning.

Section – A

1. Attempt **all** parts of this question. Each part carries **2** marks. **10 × 2 = 20**

(a) Prove that if M and N in

$M(x, y)dx + N(x, y)\, dy = 0$

Satisfy the equation

$$\frac{\partial M}{\partial y} + \frac{3}{y}M = \frac{\partial N}{\partial x},$$

then y^3 is an integrating factor.

(b) $y = (c_1 + c_2x + c_3x^2)e^x$ is the solution of the differential equation ...., c_1, c_2, c_3 are constants.

(c) Classify the singular points of the differential equation

$$x^2\frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0,$$

a and b are constants.

(d) Show that $\sum_{n=0}^{\infty} P_n(x) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1-x}}$.

(e) State the conditions for the existence of Laplace Transform.

(f) State Convolution Theorem.

(g) State Direichlet conditions for the expansion of $f(x)$ in Fourier series.

(h) Classify the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$$

(i) Explain briefly the method of separation of variables in solving a given partial differential equation.

(j) The two-dimensional wave equation is ________.

Section – B

Attempt any **three** parts of this question. Each part carries equal marks. **3 × 10 = 30**

2. (a) An R–L–C circuit connected in series has R = 90 Ohms, $C = \frac{1}{140}$ farad, L = 10 henries and an applied voltage E(t) = 10 cos t. Assuming no initial charge on the capacitor, but an initial current of 1 ampere at = 0, when the voltage is first applied, find the subsequent charge on the capacitor and the amplitude of the steady-state charge.

(b) Find the series solution, about $x = 0$, of the equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 0$, by the Frobenius method.

(c) Using convolution, solve the initial value problem

$$\frac{d^2y}{dt^2} + 9y = \sin 3t$$

given $y = 0, \frac{dy}{dt} = 0$ at $t = 0$.

(d) Find the Fourier series representation upto second harmonics of $f(x)$ which is given in the following table :

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

(e) Solve the boundary value problem

$$\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y^2} = 0,$$

satisfying the conditions $z(x, 0) = 0$, $z(x, \pi) = 0$, $z(0, y) = 4 \sin 3y$.

Section – C

Attempt any **two** parts from each question of this Section. Each part carries equal marks. **5 × 10 = 50**

3. (a) If $(x + y)^n$ is an integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$, find n and then solve the equation.

(b) Solve : $(D^2 - 3D + 2)y = \sin(e^{-x})$

(c) Solve : $\left(\frac{d}{dx} + \frac{1}{x}\right)^2 y = x^{-4}$.

4. (a) Prove that $(n + 1) P_{n+1}(x) = (2n + 1)\, x\, P_n(x) - nP_{n-1}(x)$.

(b) Show that

$$\int xJ_0^2(x)dx = \frac{1}{2}x^2\left[J_0^2(x) + J_1^2(x)\right] + c.$$

(c) Show that

$$J_3(x) = \left(\frac{8}{x^2} - 1\right) J_1(x) - \frac{4}{x} J_0(x)$$

5. (a) State second shifting theorem for Laplace transform and hence find the Laplace transform of the following function :

$$f(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$$

(b) Find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)^3}$.

(c) Show that

$$\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt = \log \frac{2}{3}$$

6. (a) Find the Fourier series of $f(x) = \frac{3x^2 - 6\pi x + 2\pi^2}{12}$ in the interval $(0, 2\pi)$. Hence, deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

(b) Find the half-range sine series of $f(x) = (lx - x^2)$ in the interval $(0, l)$. Hence, deduce that

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots$$

(c) Solve : $(D^2 + DD' - 6D'^2)z = y \sin x$

7. (a) Solve : $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$,

α constant, subject to the boundary conditions $u(0, t) = 0$, $u(\pi, t) = 0$ and the initial condition $u(x, 0) = \sin 2x$.

(b) Find the deflection of the vibrating string which is fixed at the ends $x = 0$ and $x = 2$ and the motion is started by displacing the string into the form $\sin^3\left(\frac{\pi x}{2}\right)$ and releasing it with zero initial velocity at $t = 0$.

(c) Determine the electromotive force $e(x, t)$ in a transmission line of length b, t seconds after the ends were suddenly grounded. Assume that R and G are negligible and the initial conditions are $i(x, 0) = i_0$ and $e(x, 0) = e_1 \sin \frac{\pi x}{b} + e_5 \sin \frac{5\pi x}{b}$. Here G and R denote the terms for the effect of leakage and resistance respectively.
