Printed Pages : 8   **EAS – 203**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9610   Roll No.

B. Tech.

(SEM. II) EXAMINATION, 2008-09
MATHEMATICS - II

Time : 3 Hours]   *[Total Marks : 100*

SECTION - A

Note :   Attempt all questions. All questions   **2×10=20**
             carry equal marks.

Fill up the appropriate answers in the space provided :

1   (a)   The differential equation

$$\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^2 + e^x = \sin xy \text{ is of}$$

_____ order and _____ degree.

     (b)   A particular solution of the differential equation

$$\frac{dy}{dt} = \frac{1}{2}(y^2 - 1) \text{ that satisfies the initial condition}$$

$y(0) = 2$ is _____.

     (c)   The general solution of the differential equation

$$\frac{d^5y}{dx^5} - \frac{d^3y}{dx^3} = 0 \text{ is given by } _____.$$

9610]   1   [Contd..

Pick up the Correct Answer from the following :

(d) The Rodrigues formula for Legendre Polynomial $P_n(x)$ is given by

(i) $P_n(x) = \dfrac{1}{\lfloor n \; 2^n} \dfrac{d^n}{dx^n} (x^2 - 1)^n$

(ii) $P_n(x) = \dfrac{\lfloor n}{2^n} \dfrac{dn}{dx^n} (x^2 - 1)^{n-1}$

(iii) $P_n(x) = \dfrac{\lfloor n}{2^{n-1}} \dfrac{d^n}{dx^n} (x^2 - 1)^{n-1}$

(iv) $P_n(x) = \dfrac{1}{\lfloor n \; 2^n} (x^2 - 1)^n$

(e) The Laplace transform of the function

$$f(t) = \begin{cases} 1 \;, & 0 \le t < 2 \\ -1, & 2 \le t < 4, \end{cases} \quad f(t + 4) = f(t) \text{ is given as}$$

(i) $\dfrac{1 - e^{-2s}}{s(1 + e^{-2s})}$

(ii) $\dfrac{1 + e^{-2s}}{s(1 + e^{-2s})}$

(iii) 0

(iv) $\dfrac{s+1}{s-1}$

(f) The inverse Laplace transform of $\log\left(\dfrac{s+1}{s-1}\right)$ is

given by

(i) $\dfrac{2}{t}\cosh t$

(ii) $\dfrac{2}{t}\sin h t$

(iii) $2t\cos t$

(iv) $2t\sin t$

Indicate **True / False** for the statements made therein -

(g) (i) A function $f(x)$ is even if $f(-x) = -f(x)$
            (True / False)

(ii) A function $f(x)$ is odd if $f(-x) = f(x)$.
      (True / False)

(iii) Most functions are neither even nor odd.
       (True / False)

(iv) A function $f(x)$ can always be expressed as an
      arithmetic mean of an even and odd function as

$$f(x) = \frac{1}{2}\left[f(x) + f(-x)\right] + \frac{1}{2}\left[f(x) - f(-x)\right].$$

(True / False)

(h) (i) With usual symbols, the PDE

$u_{xx} + u^2 u_{yy} = f(xy)$ is non-linear in $'u'$

and is of second order. (True / False)

(ii) The small transverse vibrations of a string are governed by one dimensional heat equation

$$y_t = a^2 y_{xx}$$ (True / False)

(iii) Two dimensional steady state heat flow is given by Laplace's equation $u_t = a^2(u_{xx} + u_{tt})$. (True / False)

(iv) The PDE of all sphere whose centre lie on z-axis and given by equations $x^2 + y^2 + (z - a)^2 = b^2$, a and b being constants, are governed by $x z_y - y z_x = 0$. (True / False)

(i) Applying the method of separation of variables techniques, the solution to the P.D.E. $3u_x + 2u_y = 0$

is _____, where $u_x = \dfrac{\partial u}{\partial x}$, $u_y = \dfrac{\partial u}{\partial y}$.

Match the column for the items of the Left side to that of right side :

(j) A second order P.D.E. in the function $'u'$ of two independent variables x, y given with usual symbols

$$Au_{xx} + Bu_{xy} + Cu_{yy} + F(u) = 0,$$ then

(i)	Hyperbolic	(a)	$B^2 - 4AC = 0$
(ii)	Parabolic	(b)	$B^2 - 4AC < 0$
(iii)	Elliptic	(c)	$B^2 - 4AC > 0$
(iv)	Not classified	(d)	$A = B = C = 0$

SECTION - B

Note : Attempt any **three** questions from this   3×10=30
          section. All questions carry equal marks.

2 (a) Apply the method of variation of parameters to
        solve the ordinary differential equations

$$\frac{d^2 y}{dx^2} + y = \tan x, \ 0 < x < \pi/2$$

   (b) Show that Bessel's function $J_n(x)$ is an even function
        when n is even and is odd function when n is odd.
        Express $J_6(x)$ in terms of $J_0(x)$ and $J_1(x)$.

   (c) Use convolution theorem to find the inverse of

        the function $\dfrac{1}{(s^2 + a^2)^2}$.

   (d) Obtain the Fourier series of $f(x) = \left(\dfrac{\pi - x}{2}\right)$ in

        the interval $(0, \ 2\pi)$ and hence deduce

        $$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

   (e) A string of Length $'L'$ is stretched and fastened to
        two fixed points. Find the solution of the wave

        equation $y_{tt} = a^2 y_{xx}$ when initial displacement is

        $y(x, \ 0) = f(x) = b \sin\left(\dfrac{\pi x}{L}\right)$, where symbols

        have usual meaning.

SECTION - C

Note :　　Attempt all questions from this section,　**[5×2]×5=50**
　　　　　　selecting any **two** parts from each question.
　　　　　　All questions carry **equal** marks.

3　　(a)　The equations of motion of a particle are given by

$$\frac{dx}{dt} + wy = 0, \quad \frac{dy}{dt} - wx = 0$$. Find the path of the

particle and show that it is a circle.

　　(b)　Integrate the differential equation

$$\frac{d^2x}{dt^2} + 2x\frac{dx}{dt} + w^2x = a\cos pt$$ and give the

physical interpretation of the complete solution.
Also deduce that the solution takes the form

$$x = \frac{a\cos(pt-\theta)}{\sqrt{(w^2-p^2)^2 + 4k^2p^2}}\left(\tan\theta = \frac{2kp}{w^2-p^2}\right) \text{ As } t \to \infty$$

　　(c)　Find the complete solution of the differential

equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\,e^x\cos x$.

4　　(a)　For the Bessel's function, prove that

$$J_{\frac{1}{2}}(x) = \left(\sqrt{\frac{2}{\pi x}}\right)\cdot\sin x$$

(b) Estimate the recurrence relation for Legendre's polynomials -

$$x P_n^{'}(x) = n P_n(x) + P_{n-1}^{'}(x)$$

(c) Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials.

5 (a) Using Laplace transformation, show that

$$\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}.$$

(b) A particle moves in a line so that its displacement x from a fixed point 0 at any time t, is given by
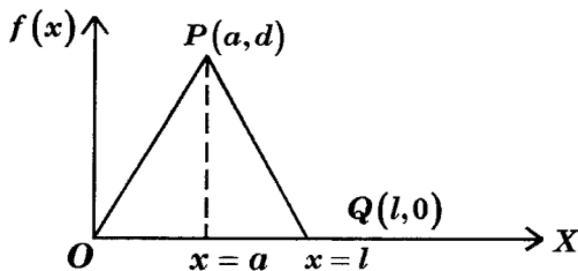
$$\frac{d^2 x}{dt^2} + 4 \frac{dx}{dt} + 5x = 80 \sin 5t.$$ Using Laplace transform, find its displacement at any time t if initially particle is at rest at $x = 0$.

(c) Find the Laplace transform of the function

$$f(t) = \begin{cases} t - 1, & 1 < t < 2 \\ 3 - t, & 2 < t < 3 \end{cases}$$

6 (a) Find the half period sine series for $f(x)$ given in the range $(0, l)$ by the graph OPQ as shown in figure.

(b)   Find the Fourier series expansion for

$$f(x) = x + \frac{x^2}{4}, \quad -\pi \leq x \leq \pi.$$

(c)   Solve the P.D.E.

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$$

7   (a)   Find the temperature in a bar of length 2 whose end are kept at zero and lateral surface is insulated if

the initial temperature is $\sin\dfrac{nx}{2} + 3\sin 5\pi\dfrac{x}{2}$.

(b)   Apply the method of separation of variables to

solve $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$.

(c)   Solve the P.D.E. by separation of variables method,

$u_{xx} = u_y + 2u, \quad u(0,\ y) = 0,$

$\dfrac{\partial}{\partial x} u(0,\ y) = 1 + e^{-3y}.$