Printed Pages : 4 **AG-121**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9920 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. II) EXAMINATION, 2006-07

MATHEMATICS : II

Time : 3 Hours] *[Total Marks : **100***

Note : *Attempt all the questions. Internal choice is mentioned for each question.*

1 Attempt any **four** parts of the following : **5×4=20**

(a) Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point $(1, 2, 3)$ in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$.

(b) Define gradient, divergence and curl. Assuming necessary condition(s) on $\vec{v}$ and $\vec{u}$ prove that $div\,(\vec{u} \times \vec{v}) = \vec{v} \cdot curl\ \vec{u} - \vec{u}\,.\ curl\ \vec{v}$.

(c) Find the work done by a force $\vec{F}(x, y, z)$ applied at a point $P(1, 2, 3)$ to displace it to point $Q(5, 1, 7)$.

(d) If f and g are two scalars functions prove that

$div\,(f\,\vec{\nabla} g) = f\,\nabla^2 g + \vec{\nabla} f \cdot \vec{\nabla} g$ where

$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.

(e) Show that the vector function

$\vec{V}(x, y, z) = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$

is solenoidal.

(f) Show that $div\,(grad\ r^n) = n(n+1)r^{n-1}$

where $r = \sqrt{x^2+y^2+z^2}$

2 Attempt any **four** parts of following : **5×4=20**

(a) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

(b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$.

(c) Find the volume of the ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(d) Evaluate the line integral

$$\int_C (xy + y^2)dx + x^2 dy$$

Where C is bounded by $y = x$ and $y = x^2$.

(e) State the Gauss divergence and verify for

$$\vec{F} = (x^2 - yz)\,\hat{i} + (y^2 - zx)\,\hat{j} + (z^2 - xy)\,\hat{k}$$

taken over the cube $0 \le x \le 1$, $0 \le y \le 1$ and $0 \le z \le 1$.

(f) State and verify the Stoke's theorem for the vector field $\overline{F} = (x^2 - y^2)\hat{i} + 2xy\,\hat{j}$ by integrating round the rectangle in plane $z = 0$ and bounded by the lines $x = 0$, $y = 0$, $x = a$ and $y = b$.

3 Attempt any **two** parts of following : **10×2=20**

(a) Solve the differential equations

(1) $(hx + by + f)\,dy + (ax + hy + g)dx = 0$

(2) $ye^y dx = (y^3 + 2xe^y)dy$

(b) Solve $\sec^2 y \dfrac{dy}{dx} + x \tan y = x^3$.

(c) Solve $(D^2 + 4D + 5)y = e^x \cos x + x^2$.

4 Attempt any **two** parts of following : **10×2=20**

(a) Prove that, for Bessel's function $J_n(x)$

$$J_n(x) = \frac{x}{2n}\left[J_{n-1}(x) + J_{n+1}(x)\right].$$

(b) For Legendre polynomial $P_n(x)$ prove that

$$\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}.$$

(c) Solve $(p^2 + q^2)y = qz$.

5 Attempt any **two** parts of the following : **10×2=20**

(a) Find the inverse of the matrix $\begin{bmatrix} 13 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 11 \end{bmatrix}$.

(b) Solve the system of linear equations :

$$x + 2y + z = 3$$
$$2x + 3y + 2z = 4$$
$$3x + 4y + 3z = 17$$

(c) State and prove Cayley-Hamilton theorem.