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Paper Id:

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Sub Code: NAS 203

Roll No.

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B TECH

(SEM- II) THEORY EXAMINATION 2017-18
ENGINEERING MATHEMATICS-II

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief. 2 x 10 = 20

- a. Find the P. I of $(D^2 - 4)y = x^2$, where $D = \frac{d}{dx}$.
- b. Find the first part of C.F in the solution of the differential equation $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - (1 - \cot x)y = e^x \sin x$.
- c. Express the polynomial $f(x) = x^2 + x + 1$ in terms of Legendre's polynomials.
- d. Show that: $J_{1/2}(x) = J_{-1/2}(x) \cot x$.
- e. Find the Laplace transform of $\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^2$.
- f. Find $L^{-1}\left\{\frac{1}{s(p-1)}\right\}$.
- g. Expand $f(x) = x$ as a half-range Sine series in $0 < x < 2$.
- h. Solve: $2p + q = 3$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.
- i. Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y} = 0$.
- j. Write the one dimensional wave equation.

SECTION B

2. Attempt any three of the following: 10 x 3 = 30

- a. Solve: $\frac{d^2x}{dt^2} + y = \sin t$, $\frac{d^2y}{dt^2} + x = \cos t$.
- b. Solve in series: $x(x-1)\frac{d^2y}{dx^2} + (3x-1)\frac{dy}{dx} + y = 0$.
- c. State convolution theorems for the inverse Laplace transform. Hence or otherwise find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)^2}$.
- d. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$. Deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi - 2}{4}$.
- e. A rod of length l with insulated sides is initially at a uniform temperature u_0 . Its ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature function $u(x, t)$.

SECTION C

3. Attempt any *one* part of the following: 10 x 1 = 10

(a) By changing the independent variable, solve the differential equation

$$x \frac{d^2 y}{dx^2} + (4x^2 - 1) \frac{dy}{dx} + 4x^3 y = 2x^3.$$

(b) Solve by the method of variation of parameters: $\frac{d^2 y}{dx^2} + a^2 y = \sec ax.$

4. Attempt any *one* part of the following: 10 x 1 = 10

(a) Prove that: $P_n(x) = \frac{1}{2^n n!} \frac{d^n (x^2 - 1)^n}{dx^n}.$

(b) Prove that: $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{(3 - x^2)}{x^2} \sin x - \frac{3 \cos x}{x} \right].$

5. Attempt any *one* part of the following: 10 x 1 = 10

(a) Draw the graph and find the Laplace transform of the triangular wave function

$$\text{of period } 2c \text{ given by } f(t) = \begin{cases} t, & 0 < t \leq c \\ 2c - t, & c < t < 2c \end{cases}.$$

(b) Find the inverse Laplace transform of $\frac{s}{(s^2 + 1)(s^2 + 4)}.$

6. Attempt any *one* part of the following: 10 x 1 = 10

(a) Obtain a half range cosine series for $\begin{cases} kx, & 0 < x < l/2 \\ k(l-x), & l/2 \leq x \leq l \end{cases}.$

Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

(b) Solve the partial differential equation $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2),$
where $p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}.$

7. Attempt any *one* part of the following: 10 x 1 = 10

(a) Use the method of separation of variable to solve the equation $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u,$
given that $u(x, 0) = 6e^{-3x}.$

(b) An infinitely long plane uniform plate is bounded by two parallel edges and end at right angles to them. The breadth is $\pi.$ This end is maintained at temperature u_0 at all points and the other edges are kept at zero temperature. Determine the temperature at any point of the plate in steady state.