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B TECH

(SEM II) THEORY EXAMINATION 2017-18

ENGINEERING MATHEMATICS II

Time: 3 Hours *Total Marks: 100*

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

a. Find the particular integral of $(4D^2+4D-3)y = e^{2x}$, where $D \equiv \frac{d}{dx}$

b. Solve the differential equation :
$\frac{d^2y}{dx^2}+y=0$; given that $y(0)=2$ and $y\left(\frac{\pi}{2}\right)=-2$

c. For Bessel's function $J_n(x)$, find the value of $J_{-\frac{1}{2}}(x)$.

d. Express x^3 in terms of Legendre's polynomials.

e. Find the Laplace transform of t^3e^{-3t}.

f. State change of scale property of Laplace transform.

g. Solve the following partial differential equation: $\frac{\partial^2 z}{\partial x^2}+4\frac{\partial^2 z}{\partial x\partial y}-5\frac{\partial^2 z}{\partial y^2}=0$

h. Find P.I. of the following partial differential equation: $4\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=e^{x+2y}$

i. Write the Laplace equation in two dimensions.

j. Classify the following partial differential equation as hyperbolic, parabolic or elliptic: $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial t}-2\frac{\partial^2 u}{\partial t^2}=0$

SECTION B

2. Attempt any *three* of the following: 10 x 3 = 30

a. Solve the following simultaneous differential equations
$\frac{dx}{dt}=-wy, \frac{dy}{dt}=wx$.
Also show that the point (x,y) lies on a circle.

b. For Bessel's function $J_n(x)$, prove that $xJ_n'(x)=n\,J_n(x)-x\,J_{n+1}(x)$

c. Find the Laplace transform of the following periodic function with period $\frac{2\pi}{w}$:

$$F(t)=\begin{cases}\sin wt & ,0<t\le\frac{\pi}{w}\\ 0, & \frac{\pi}{w}<t<\frac{2\pi}{w}\end{cases}$$

d. Obtain the Fourier series for the function $f(x)=x^2, -\pi\le x\le\pi$. Also show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots.=\frac{\pi^2}{6}$

e. Using the method of separation of variables, solve
$\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$. Given that $u(x,0)=6e^{-3x}$

SECTION C

3. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) Apply the method of variation of parameters to solve the following differential equation: $\frac{d^2y}{dx^2} + y = \tan x$

(b) Solve: $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$

4. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) Solve $(1-x^2)y'' - xy' + 4y = 0$ in series in powers of x.

(b) State and prove Rodrigue's formula for Legendre's polynomials.

5. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) Express the following function in terms of unit step function and find its Laplace transform: $F(t) = \begin{cases} 2 & if\ 0 < t < \pi \\ 0 & if\ \pi < t < 2\pi \\ sint & if\ t > 2\pi \end{cases}$

(b) Evaluate by using convolution theorem: $L^{-1}\left\{\frac{1}{p(p^2-a^2)}\right\}$

6. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) Find the Fourier half range cosine series for the function:

$$f(x) = \begin{cases} 1 & , 0 < x < \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} < x < \pi \end{cases}$$

(b) Find the general solution of $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ where $p \equiv \frac{\partial z}{\partial x}$ and $q \equiv \frac{\partial z}{\partial y}$.

7. **Attempt any *one* part of the following:** **10 x 1 = 10**

(a) The vibrations of an elastic string are governed by the partial differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$. The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string for $t > 0$.

(b) Find the temperature in a bar of length l, whose ends $x = 0$ and $x = l$ are kept at zero temperature, if the initial temperature of the bar is $f(x)$.