Printed Pages : 2　　　　**Roll No.** ☐☐☐☐☐☐☐☐☐☐☐☐☐　　**EAS203**

B.TECH.
THEORY EXAMINATION (SEM–II) 2016-17
ENGINEERING MATHEMATICS - II

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　　　*Max. Marks : 100*

Note : *Be precise in your answer. In case of numerical problem assume data wherever not provided.*

SECTION – A

1.　**Explain the following:**　　　　　　　　　　　　　　　　　　　**10 x 2 = 20**

　　(a)　Show that the differential equation $y\, dx - 2x\, dy = 0$ represents a family of parabolas.

　　(b)　Classify the partial differential equation

$$(1 - x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial y \partial x} + (1 - y^2)\frac{\partial^2 z}{\partial y^2} = 2z$$

　　(c)　Find the particular integral of $(D - \alpha)^2 y = e^{\alpha x} f''(x)$.

　　(d)　Write the Dirichlet's conditions for Fourier series.

　　(e)　Prove that $J'_0(x) = -J_1(x)$.

　　(f)　Prove that $L\left[e^{at}f(t)\right] = F(s - a)$

　　(g)　Find the Laplace transform of $f(t) = \frac{\sin at}{t}$.

　　(h)　Write one and two dimensional wave equations.

　　(i)　Find the constant term when $f(x) = |x|$ is expanded in Fourier series in the interval (-2, 2).

　　(j)　Write the generating function for Legendre polynomial $P_n(x)$.

SECTION – B

2.　**Attempt any five of the following questions:**　　　　　　　　**5 x 10 = 50**

　　(a)　Solve the differential equation

$$(D^2 + 2D + 2)y = e^{-x}\sec^3 x, \qquad where\ D = \frac{d}{dx}.$$

　　(b)　Prove that $(n + 1)P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x),$
　　　　where $P_n(x)$ is the Legendre's function.

　　(c)　Find the series solution of the differential equation

$$2x^2\frac{d^2 y}{dx^2} + x\frac{dy}{dx} - (x + 1)y = 0.$$

　　(d)　Using Laplace transform, solve the differential equation

$$\frac{d^2 y}{dt^2} + 9y = \cos 2t\ ;\ \ y(0) = 1\ ,\ y\left(\frac{\pi}{2}\right) = -1.$$

　　(e)　Obtain the Fourier series of the function,

$$f(t) =\ \ t\ ,\ \ -\pi < t < 0$$
$$=\ -t\ ,\ \ 0 < t < \pi.$$

　　　Hence, deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ \dots\dots\dots\dots\dots\ = \frac{\pi^2}{8}$

　　(f)　Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions $u(0,\ y) = 0,$

$$u(l, y) = 0, u(x, 0) = 0 \text{ and } u(x, a) =\ \sin\frac{n\pi x}{l}.$$

　　(g)　Solve the partial differential equation:

$$(D^3 - 4D^2 D' + 5D\,D'^2 - 2D'^3)z = e^{y+2x} + \sqrt{y + x}$$

　　(h)　Using convolution theorem find $L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$

SECTION – C

Attempt any two of the following questions: **2 x 15 = 30**

3. **(a)** Solve the differential equation $(D^2-2D+1)y = e^x \sin x$

   (b) Solve the equation by Laplace transform method:
$$\frac{dy}{dt} + 2y + \int_0^t y \, dt = \sin t, \quad y(0) = 1.$$

   (c) Solve the partial differential equation
$$(y^2 + z^2)\, p - xyq + zx = 0, where \; p = \frac{\partial z}{\partial x} \; \& \; q = \frac{\partial z}{\partial y}$$

4. **(a)** Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.

   (b) Express $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre's polynomial.

   (c) Expand $f(x) = 2x - 1$ as a cosine series in $0 < x < 2$.

5. **(a)** Show that $J_3(x) = \left(\frac{8}{x^2} - 1\right)J_1(x) - \frac{4}{x}J_0(x)$.

   (b) Solve the $2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} + 5z = 0;\; z(0, y) = 2e^{-y}$ by the method of separation of variables.

   (c) A tightly stretched string with fixed end $x = 0$ and $x = l$ is initially in a position given by $y = a\sin\frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x, t)$.