Printed Pages:2 **Sub Code: KAG 201**

Paper Id: 180272 **Roll No.** | | | | | | | | | | |

B TECH
(SEM II) THEORY EXAMINATION 2018-19
BASIC MATHEMATICS II

Time: 3 Hours ***Total Marks: 100***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

a. Define trivial and non-trivial solution.

b. Find the rank of A=$\begin{bmatrix} 16 & 4 & 12 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix}$.

c. State the Cauchy's Riemann equations in polar form.

d. Determine whether $f(z) = \frac{1}{z}$ is analytic or not.

e. State the Green's theorem.

f. Find the value of (grad r)where$\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$.

g. Solve $\left(2D_x^2 + 5D_xD_y + 2D_y^2\right)z = 0$.

h. Find the half range Fourier sine series for$f(x) = x\ in\ -\pi < x < \pi$.

i. Write the one dimensional heat equation in unsteady state.

j. Solve $u_{xx} - u_y = 0$ by separation of variables.

SECTION B

2. Attempt any *three* of the following: **10x3=30**

a. Find P and Q such that the normal form of A=$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 3 & 1 & 2 \end{bmatrix}$ is PAQ. Hence, find the rank.

b. Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

c. Evaluate$\nabla^2 r^m$where $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$.

d. Find the Fourier series of $f(x) = xcos\left(\frac{\pi x}{l}\right) in - l \le x \le l.$

e. Solve Laplace equation in a rectangle in the $0 < x < m\ and\ 0 < y < n$ satisfying the following boundary conditions u(x,0)=0,u(x,n)=0,u(0,y)=0 and u(m,y)=ky(n-y)

SECTION C

3. Attempt any *one* part of the following: 10x1=10

a. Find A^{-1}, A^{-2} if A=$\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.

b. Show that A=$\begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is skew-hermitian and also unitary. Find the eigen values and eigen vectors.

4. Attempt any *one* part of the following: 10x1=10

a. If f (z) is regular function of z, show that$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2$.

b. Show that the function defined by $f(z)=\sqrt{|xy|}$ is not regular at the origin, although the Cauchy-Riemann equations are satisfied there.

5. Attempt any *one* part of the following: 10x1=10

a. Prove that $(\vec{A}) = (6xy+z^3)\vec{\imath}+(3x^2-z)\vec{j}+(3xz^2-y)\vec{k}$ is irrational. Find a scalar function $(x,y,z) such\ that\ \vec{A}=\nabla f$.

b. Find the flux of the vector field $\vec{A}=(x-2z)\vec{\imath}+(x+3y+z)\vec{j}+(5x+y)\vec{k}$ through the upper side of the triangle ABC with vertices at the points A(1,0,0),B(0,1,0) and C(0,0,1).

6. Attempt any *one* part of the following: 10x1=10

a. Find the Fourier series expansion of $f(x)=xsinx\ in -\pi<x<\pi$.

b. Solve the partial differential equation $z_{xx}+z_{yy}=x^2y^2$.

7. Attempt any *one* part of the following: 10x1=10

a. Solve the following PDE by the method of separation of variables: $3u_x+2u_y=0\ with\ u(x,0)=6e^{-x}$.

b. Derive the general solution of one dimensional wave equation .