Printed Pages : 8 **EAS-103/ASM-101**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601 Roll No. | | | | | | | | | | |

B. Tech.

(Semester-I) Theory Examination, 2011-12

MATHEMATICS-I/ENGINEERING MATHEMATICS-I

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as indicated. The symbols have their usual meaning.

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. 2×10=20

1. (a) Find y_n if $y = \dfrac{x^n - 1}{x-1}$.

(b) What is the asymptote of the curve $y^2(2a-x) = x^3$?

(c) If $u = x^2yz - 4y^2z^2 + 2xz^3$, find the value of :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}.$$

(d) Calculate :

$$\frac{\partial(u,v)}{\partial(x,y)}$$

for $x = e^u \cos v$ and $y = e^u \sin v$.

(e) Find the value of :

$$\Gamma\left(-\frac{5}{2}\right).$$

(f) Find the value of the integral $\iint_R xy\,dx\,dy$, where R is the region bounded by the x-axis, the line $y = 2x$ and the parabola $x^2 = 4ay$.

(g) Show that the vector :

$$\vec{V} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$$

is solenoidal.

(h) State Green's theorem for a plane region.

(i) The matrix :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

satisfies the matrix equation $A^3 - 6A^2 + 11A - I = 0$, where I is an identity matrix of order 3. Find A^{-1}.

(j) Show that the matrix :

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

is diagonalizable.

Section-B

Attempt any *three* parts of this question. 10×3=30

2. (a) If $y(x) = \sin px + \cos px$, prove that :

$$y_n(x) = p^n[1+(-1)^n \sin 2px]^{1/2}.$$

Hence, show that $y_8(\pi) = \left(\frac{1}{2}\right)^{31/2}$ when $p = \frac{1}{4}$.

(b) Evaluate :

$$[(3.82)^2 + 2(2.1)^3]^{1/5}$$

using theory of approximation.

(c) Evaluate :

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\, dx\, dy.$$

(d) Evaluate :

$$\iint_S \vec{F} \cdot \hat{n}\, dS,$$

where $\vec{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x+3y+6z=12$ in the first octant.

(e) If :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

and adj (adj A) $= A$, find a.

Section-C

All questions of this Section are compulsory. Attempt any *two* parts from each question. 10×5=50

3. (a) If $y = \left(\frac{1+x}{1-x}\right)^{1/2}$, prove that :

$$(1-x^2)y_n - [2(n-1)x+1]\, y_{n-1} - (n-1)(n-2)y_{n-2} = 0.$$

(b) Obtain the series for $\log_e(1+x)$ and then find the series for $\log_e\left(\frac{1+x}{1-x}\right)$ and hence, determine the value of $\log_e\left(\frac{11}{9}\right)$ up to five places of decimal.

(c) Find the asymptotes of:

$$y^3-x^2y-2xy^2+2x^3-7xy+3y^2+2x^2+2x+2y+1=0.$$

4. (a) If:

$$u=\sin^{-1}\left(\frac{x^{1/3}+y^{1/3}}{x^{1/2}-y^{1/2}}\right)^{1/2},$$

show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{1}{12}\tan u.$

(b) Are the functions :

$$u=\frac{x-y}{x+z},\quad v=\frac{x+z}{y+z}$$

functionally dependent ? If so, find the relation between them.

(c) Using the Lagrange's method, find the maximum and minimum distances from the origin to the curve $3x^2+4xy+6y^2=140$.

5. (a) Prove that :

$$B(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx.$$

(b) Find the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the coordinate planes.

(c) Find the volume of the solid which is bounded by the surfaces $2z=x^2+y^2$ and $z=x$.

6. (a) If $\vec{F}=(\vec{a}.\vec{r})\vec{r}$, where $\vec{a}$ is a constant vector, find curl $\vec{F}$ and prove that it is perpendicular to $\vec{a}$.

(b) Find the work done in moving a particle in the force field :

$$\vec{F}=3x^2\hat{i}+(2xz-y)\hat{j}+z\hat{k}$$

along the curve $x^2=4y$ and $3x^3=8z$ from $x=0$ to $x=2$.

(c) Prove that :

$$\iint_S \frac{1}{\sqrt{a^2x^2+b^2y^2+c^2z^2}}\,dS=\frac{4\pi}{\sqrt{abc}},$$

where S is the ellipsoid $ax^2+by^2+cz^2=1$.

7. (a) Find the value of P for which the matrix :

$$A=\begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$$

is be of rank 1.

(b) Show that the system of equations :

$$3x+4y+5z=A$$

$$4x+5y+6z=B$$

$$5x+6y+7z=C$$

are consistent only if A, B and C are in arithmetic progression (A. P.).

(c) **Show that :**

$$A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

is Skew-Hermitian and also unitary.