(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2012-13

MATHEMATICS—I

Time : 3 Hours *Total Marks : 100*

SECTION—A

1. All parts for this question are compulsory : **(2×10=20)**

(a) Find the 8^{th} derivative of x^2e^x.

(b) If $x^2 = au + bv$, $y^2 = au - bv$, then find $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v$.

(c) Find the stationary points of

$$f(x, y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1.$$

(d) If $x = u(1 + v)$, $y = v(1 + u)$, then find the Jacobian of u, v with respect to x, y.

(e) Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into normal form.

(f) Prove that the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.

(g) Evaluate $\int_0^1 \int_0^{x^2} x\, e^y \, dxdy$.

(h) Evaluate $\Gamma(-3/2)$.

(i) Find the value of m if $\vec{F} = mx\hat{i} - 5y\hat{j} + 2z\hat{k}$ is a solenoidal vector.

(j) Find the unit normal at the surface $z = x^2 + y^2$ at the point (1, 2, 5).

SECTION—B

2. Attempt any **three** parts of the following : **(3×10=30)**

(a) If $y = \left(x + \sqrt{1+x^2}\right)^m$, then find the n^{th} derivative of y at $x = 0$.

(b) Find the maximum and minimum distance of the point (1, 2, –1) from the sphere $x^2 + y^2 + z^2 = 24$.

(c) Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$

(d) Evaluate $\iiint_V (ax^2 + by^2 + cz^2)\,dxdydz$ where V is the region bounded by $x^2 + y^2 + z^2 \leq 1$.

(e) Verify Gauss's divergence theorem for the function $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ over unit cube.

SECTION—C

Attempt any **two** parts from each question of this section. All questions are compulsory. **[(2×5)×5=50)]**

3. (a) State and prove Euler's theorem for homogeneous functions.

(b) Expand $f(x, y) = e^x \tan^{-1} y$ in powers of $(x - 1)$ and $(y - 1)$ upto two terms of degree 2.

(c) If $z = f(x, y)$ where $x = e^u \cos v$, $y = e^u \sin v$, prove that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2\right].$$

4. (a) If $x + y + z = u$, $y + z = uv$, $z = uvw$, then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

(b) The two sides of a triangle are measured as 50 cm and 70 cm, and the angle between them is 30°. If there are possible errors of 0·5% in the measurement of the sides and 0·5 degree in that of the angle, find the maximum approximate percentage error in measuring the area of the triangle.

(c) Show that $u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$ are not independent. Find the relation between them.

5. (a) Test the consistency and hence, solve the following set of equations :

$$\begin{aligned} 10y + 3z &= 0, \\ 3x + 3y + 2z &= 1, \\ 2x - 3y - z &= 5, \\ x + 2y &= 4. \end{aligned}$$

(b) Using elementary transformations, find the rank of the following matrix :

$$A = \begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$

(c) Examine the following vectors for linearly dependent and find the relation between them, if possible :

$$X_1 = (1, 1, -1, 1),\ X_2 = (1, -1, 2, -1),\ X_3 = (3, 1, 0, 1).$$

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6. (a) Prove that : $\sqrt{\pi}\,\Gamma(2n) = 2^{2n-1}\,\Gamma(n)\,\Gamma\left(n+\frac{1}{2}\right)$,

where n is not a negative integer or zero.

(b) Change the order of integration and hence evaluate

$$\int_0^\infty \int_0^y y\, e^{-y^2/x}\, dxdy.$$

(c) Find the area of the region occupied by the curves $y^2 = x$ and $y^2 = 4 - x$.

7. (a) Show that the vector field $\vec{F} = yz\hat{i} + (zx+1)\hat{j} + xy\hat{k}$ is conservative. Find its scalar potential. Also find the work done by $\vec{F}$ in moving a particle from (1, 0, 0) to (2, 1, 4).

(b) Prove that :

$$\text{Curl}(\vec{F} \times \vec{G}) = \vec{F}\,\text{div}\,\vec{G} - \vec{G}\,\text{div}\,\vec{F} + (\vec{G}\cdot\nabla)\vec{F} - (\vec{F}\cdot\nabla)\vec{G}.$$

(c) If $\text{div}\left[f(r)\vec{r}\right] = 0$, where $\vec{r}$ is the position vector of a point (x, y, z), then find f(r).