

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9916 Roll No.

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B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2012-13

MATHEMATICS—I

Time : 3 Hours

Total Marks : 100

Note :—(1) Attempt *all* questions.

(2) All questions carry equal marks.

(3) The symbols have their usual meaning.

1. Attempt any **TWO** parts of the following : (10×2=20)

(a) Find the rank of the following matrix by reducing it to normal form :

~~$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$~~

(b) Investigate, for what values of λ and μ do the system of equations

$$x + y + z = 6,$$

$$x + 2y + 3z = 10,$$

$$x + 2y + \lambda z = \mu$$

have :

(i) no solution

(ii) unique solution

(iii) infinite solutions ?

(c) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Hence compute A^{-1} .

2. Attempt any **FOUR** parts of the following : (5×4=20)

(a) If $y = (\sin^{-1} x)^2$, prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$$

(b) If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, prove that

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}.$$

(c) If $u = \log_e \left(\frac{x^4 + y^4}{x + y} \right)$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3.$$

(d) Trace the curve :

$$y^2(a + x) = x^2(3a - x).$$

(e) If $V = f(x^2 + 2yz, y^2 + 2xz)$, find the value of

$$\left(\frac{\partial}{\partial x} - xz \right) \frac{\partial V}{\partial x} + (x^2 - yz) \frac{\partial V}{\partial y} + (z^2 - xy) \frac{\partial V}{\partial z}$$

(f) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem.

3. Attempt any **TWO** parts of the following : (10×2=20)

(a) (i) Verify the chain rule for Jacobians, if

$$x = u, y = u \tan v, z = w.$$

(ii) If $u = x + 2y + z, v = x - 2y + 3z$ and

$$w = 2xy + xz + 4yz - 2z^2, \text{ show that they are not independent. Find the relation between } u, v \text{ and } w.$$

(b) If Δ is the area of a triangle, prove that the error in Δ resulting from a small error in C is given by

$$\delta \Delta = \frac{\Delta}{4} \left[\frac{1}{s-a} + \frac{1}{s-b} - \frac{1}{s-c} \right] \delta C.$$

(c) Find the minimum distance from the point $(1, 2, 0)$ to the cone $z = x^2 + y^2$.

4. Attempt any **FOUR** parts of the following : (5×4=20)

(a) Evaluate $\iiint_R (x - 2y + z) dz dy dx$, where R is the region

$$0 \leq x \leq 1$$

(b) Find the area lying between the parabola $y = 4x - x^2$, and the line $y = x$.

(c) Evaluate by changing the order of integration :

$$\int_0^1 \int_0^y e^x dx dy.$$

(d) Define Gamma and Beta functions. Prove that :

$$\beta(m, n) \beta(m, n) = \beta(m+n, p) \frac{\Gamma(m) \Gamma(n) \Gamma(p)}{\Gamma(m+n+p)}.$$

(e) Express the following integral in terms of Beta function

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dx}{\sqrt{1-x^2}}$$

- (f) Apply Dirichlet's integral to find the volume of the solid bounded by the co-ordinate planes and the surface

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} + \sqrt{\frac{z}{c}} = 1.$$

5. Attempt any **TWO** parts of the following : (10×2=20)

(a) (i) If $\vec{F}(x, y, z) = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$,

find curl of

- (ii) Find the directional derivative of in the direction

of \vec{r} , where

- (b) Find the circulation of round the curve C, where

and C is the rectangle whose

vertices are (0, 0), (1, 0), (1, $\pi/2$) and

- (c) State the Green's theorem. Apply this theorem to evaluate :

$$\int_C (x^2 + xy)dx + (x^2 + y^2)dy,$$

where C is the square formed by the lines $y = \pm 1$,
 $x = \pm 1$.