Printed Pages : 4 TAS104

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9916 Roll No.

B.Tech

(SEM I) ODD SEMESTER THEORY EXAMINATION 2009-10

MATHEMATICS I

Time : 3 Hours] *[Total Marks : 100*

Note : *Attempt all questions.*

1 Attempt any two parts of the following :

(a) Reduce the matrix :

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$$

to column echelon form and find its rank.

(b) Verify the Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$$

and hence find A^{-1}.

(c) Find the eigenvalues and the corresponding eigen vectors of the matrix

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

2 Attempt any **two** parts of the following :

(a) If $y=\left(x^2-1\right)^n$, prove that

$$\left(x^2-1\right)y_{n+2}+2xy_{n+1}-n(n+1)y_n=0$$

(b) If $u=x^3+y^3+z^3+3xyz$, show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=3u$$

(c) If $u=f(r)$ where $r^2=x^2+y^2$, prove that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$$

3 Attempt any **two** parts of the following :

(a) If $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,$ $z=r\cos\theta$, show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}=r^2\sin\theta$$

(b) Determine the points where the function

$$f(x, y) = x^3 + y^3 - 3xy$$

has a maximum or minimum.

(c) A rectangular box open at the top is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

4 Attempt any **two** parts of the following :

(a) Evaluate

$$\iint_A xy\, dx\, xy$$

where A is the domain bounded by $x - axis$, ordinate $x = 2a$ and the curve $x^2 = 4ay$.

(b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

(c) Evaluate :

$$\iiint_R (x + y + z)\, dx\, dy\, dz, \quad \text{where}$$

$R: 0 \le x \le 1,\ 1 \le y \le 2,\ 2 \le z \le 3$

5 Attempt any **two** parts of the following :

(a) Find a unit normal vector $\hat{n}$ of the cone of revolution $z^2 = 4\left(x^2 + y^2\right)$ at the point $P: (1, 0, 2)$.

(b) Using Green's theorem evaluate

$$\int_C \left(x^2 + xy\right)dx + \left(x^2 + y^2\right)dy$$

where C is the square formed by the lines $y = \pm 1,\ x = \pm 1$

(c) Verify Stoke's theorem for

$\vec{F} = xy^2\,\hat{i} + y\,\hat{j} + z^2 x\,\hat{k}$ for the surface of a rectangular lamina bounded by

$x = 0,\ y = 0,\ x = 1, y = 2,\ z = 0$