Sub Code:NBC101

Paper Id: 214121 **Roll No:**

B. TECH
(SEM-I) THEORY EXAMINATION 2019-20
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

a.	Find $y_n(x)$, if $y = x\sin x$.
b.	If $u = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x+y}}\right)$, then find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$.
c.	If $u = x(1-y), v = xy$, find $\dfrac{\partial(u,v)}{\partial(x,y)}$.
d.	If $x + y = 2$ then find maximum value of xy.
e.	Find the characteristic values of the matrix $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$.
f.	Find the rank and nullity of the linear transformation $f : R^2 \to R^3$ defined by $f(x,y) = (x+y, y, 0)$.
g.	Evaluate $\int_0^\infty x^{1/4} e^{-\sqrt{x}}\, dx$
h.	Change the order of integration $\int_0^1\int_1^2 \exp(x+y)dxdy$.
i.	Find the directional derivative of $f(x,y,z) = xy^2 + yz^3$ at (2, 1, -1) in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$.
j.	State Green's theorem.

SECTION B

2. Attempt any *three* of the following: 10x3=30

a.	For $y = (\sin^{-1} x)^2$ determine $y_n(0)$.
b.	Find the shortest and longest distance from the point (1, 2, -1) to the sphere $x^2 + y^2 + z^2 = 24$ using Lagrange's method.
c.	Use modal matrix to diagonalize the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$
d.	Compute the area and mass contained in first quadrant enclosed by the curve $$\left(\frac{x}{a}\right)^\alpha + \left(\frac{y}{b}\right)^\beta = 1,$$ where $\alpha, \beta > 0$. Given that density at any point is $k\sqrt{xy}$.
e.	Verify Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where surface S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.

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SECTION C

3. Attempt any *one* part of the following: 10x1=10

a.	Expand $e^x \log(1+x)$ in powers of x up to terms of third degree using Maclaurin's theorem.
b.	Trace the curve $r^2 = a^2 \cos 2\theta$.

4. Attempt any *one* part of the following: 10x1=10

a.	If $u = x + y + z$; $v = x^2 + y^2 + z^2$; $w = x^3 + y^3 + z^3 - 3xyz$, prove that u, v, w are not independent and hence find the relation among them.
b.	Compute an approximate value of $\left[(3.82)^2 + 2(2.1)^3\right]^{1/5}$.

5. Attempt any *one* part of the following: 10x1=10

a.	Test the consistency of the system of equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$ and $x - y + z = -1$ if consistent then solve it completely.
b.	Verify Cayley-Hamilton theorem and hence find the inverse of matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$.

6. Attempt any *one* part of the following: 10x1=10

a.	Find the area of the region bounded by $y^2 = 4 - x$ and $y^2 = x$.
b.	Show that $\beta(m,n) = \int_0^1 \frac{(x^{m-1} + x^{n-1})}{(1+x)^{m+n}} dx$.

7. Attempt any *one* part of the following: 10x1=10

a.	Determine the value of n for which the vector $r^n \vec{r}$ is both solenoidal and irrotational.
b.	Evaluate $\iint_S \vec{F}.\hat{n} ds$ where $\vec{F} = 6z\hat{i} - 4\hat{j} + y\hat{k}$ and S is the portion of the plane $2x + 3y + 6z = 12$ which is in the first octant.