



Printed Pages : 8

EAS103

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601

Roll No.

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B.Tech

(SEM I) ODD SEMESTER THEORY EXAMINATION 2009-10
MATHEMATICS-I

Time : 3 Hours]

[Total Marks : 100

SECTION - A

All parts of this question are compulsory.

2×10=20

1 (a) If $f(x) = f(0) + k f_1(0) + \frac{k^2}{L^2} f_2(\theta k)$, $0 < \theta < 1$

then the value of θ when $k=1$ and $f(x) = (1-x)^{5/2}$ is given as _____.

(b) The shortest distance from the point (1, 2, -1) to the sphere $x^2 + y^2 + z^2 = 24$ shall be

_____.

(c) The Jacobian $J\left(\frac{u, v}{x, y}\right)$ for $u = e^x \sin y$,

 $v = x \log \sin y$ shall be _____.

- (d) For the curve $ay^2 = x^2(a - x)$, which of the following statement(s) is/are Incorrect ?
- (i) Curve passes through origin
 - (ii) Curve is symmetrical about y axis
 - (iii) Curve has two branches
 - (iv) Curve has no tangents at origin.
- (e) If P and Q are non-singular matrices, then for Matrix ' M ', which of the following statements are correct ?
- (i) $\text{Rank}(PMQ) > \text{Rank } M$
 - (ii) $\text{Rank}(PMQ) = \text{Rank } M$
 - (iii) $\text{Rank}(PMQ) < \text{Rank } M$
 - (iv) $\text{Rank}(PMQ) = \text{Rank } M + \text{Rank}(PQ)$
- (f) If λ is an eigen value of the matrix ' M ' then for the matrix $(M - \lambda I)$, which of the following statement(s) is/are correct ?
- (i) Skew symmetric
 - (ii) Non singular
 - (iii) Singular
 - (iv) None of these.

Indicate **True / False** for the following statements :

(g) For $\int_0^{\infty} \int_x^{\infty} f(x, y) dx dy$, the change of order of integration is

- (i) $\int_0^{\infty} \int_0^{\infty} f(x, y) dx dy$ True / False

(ii) $\int_0^{\infty} \int_0^{\infty} f(x, y) dx dy$ True / False

(iii) $\int_0^{\infty} \int_0^y f(x, y) dx dy$ True / False

(iv) $\int_0^{\infty} \int_0^x f(x, y) dx dy$ True / False

(h) The value of $\sqrt{-\frac{1}{2}}$ is given by

(i) $\sqrt{\pi}$ True / False

(ii) $2\sqrt{\pi}$ True / False

(iii) $-\sqrt{\pi}$ True / False

(iv) $-2\sqrt{\pi}$ True / False

Pick up the correct option from the following :

(i) If \vec{F} is the velocity of a fluid particle then $\int_C \vec{F} \cdot d\vec{r}$

represents

(i) Work done

(ii) Circulation

(iii) Flux

(iv) Conservative field.

(j) The value of $\iint_S \vec{F} \cdot \vec{n} \, d\vec{s}$ where

$\vec{F} = ax\hat{i} + by\hat{j} + cz\hat{k}$; a, b, c being constants
is given by

(i) $\frac{\pi}{3}(a+b+c)$

(ii) $\frac{4\pi}{3}(a+b+c)$

(iii) $2\pi(a+b+c)$

(iv) $\pi(a+b+c)$.

SECTION - B

Attempt any **three** parts of the following :

10×3=30

2 (a) Determine the values of ' a ' and ' b ' for which the following system of equations has

$$3x + 5y - az = 7,$$

$$x - by + 4z = -3,$$

$$ax + 4y - 5z = 4$$

(i) No solution

(ii) A unique solution

(iii) Infinite no. of solutions.

(b) Find the value of $D^n \{x^{n-1} \log x\}$, $D^n \equiv \frac{d^n}{dx^n}$.

(c) If $u = \frac{(x+y)}{z}$, $v = \frac{(y+z)}{x}$, $w = \frac{y(x+y+z)}{(xz)}$

then show that u , v , w are not independent and find the relation between them.

(d) A rigid body is rotating with constant angular velocity ω about a fixed axis. If ' v ' is the linear velocity of any point of the body then prove that $\text{curl } v = 2\omega$.

(e) Assuming $\sqrt{n} \sqrt{1-n} = \pi \operatorname{cosec} n\pi$, $0 < n < 1$, show

$$\text{that } \int_0^{\infty} \frac{x^{p-1}}{1+x} dx = \left(\frac{\pi}{\sin p\pi} \right); \quad 0 < p < 1.$$

SECTION - C

All questions of this section are compulsory. Attempt $10 \times 5 = 50$ any two parts from each question :

3 (a) If $x = \sin \left(\frac{\log y}{a} \right)$ then evaluate the value

$$(1-x^2)y_{n+2} - (2n+1)x y_{n+1} - (n^2+a^2)y_n = 0$$

with usual symbols.

(b) If $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0.$$

(c) Verify Euler's theorem for $z = \frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}$.

4 (a) In a plane ΔABC , find the maximum value of $\cos A \cos B \cos C$.

(b) If $x = e^u \sec u$, $y = e^u \tan u$ then evaluate

$$\frac{\partial(x, y)}{\partial(u, v)}$$

$\frac{180-180}{180} = 0$

(c) The power ' P ' required to propel a steamer of length ' l ' at a speed ' u ' is given by $P = \lambda u^3 l^3$ where λ is constant. If u is increased by 3% and l is decreased by 1%, find the corresponding increase in ' P '.

5 (a) Show that row vectors of the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \text{ are linearly independent.}$$



- (b) Find the rank of the following matrix using the

elementary transformations
$$\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$$

(c) Express the matrix $A \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$

as a sum of Hermitian and Skew Hermitian matrix.

- (a) Interpret the physical meaning of $\text{curl } \vec{F}$ and $\text{div } \vec{F}$.

- (b) Verify the divergence theorem for the function

$$\vec{F} = 4xz \hat{i} - y^2 \hat{j} + yz \hat{k}; \text{ taken over the cube}$$

bounded by planes $x = 0, x = 1;$

$$y = 0, y = 1; z = 0, z = 1.$$

- (c) If a vector field is given by

$$\vec{F} = (x^2 - y^2 + x) \hat{i} - (2xy + y) \hat{j}. \text{ Is this field irrotational? If so, find its scalar potential.}$$

(a) Evaluate $\iint_R \left(1 - \frac{x^2}{a^2} + \frac{y^2}{b^2} \right) dx dy$ over the first

$\frac{3}{8} = \left(\frac{1}{8} \right)$ quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

- (b) Find the mass of the region bounded by ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1; \text{ if the density varies as the}$$

square of the distance from the centre.

- (c) A triangular prism is formed by the planes whose equations are $ay = bx$, $y = 0$ and $x = a$. Find the volume of this prism between the plane $z = 0$ and the surface $z = c + xy$.