

Printed Pages : 3



EAS-103

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199109

Roll No.

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B. Tech.

(SEM. I) (ODD SEM.) THEORY
EXAMINATION, 2014-15
MATHEMATICS - I

Time : 3 Hours]

[Total Marks : 100

Note : Attempt **all** questions. All questions carry **equal** marks.**1** Attempt any **two** parts :

a) Find the n^{th} derivative of $\frac{(2x+1)}{(2x-1)(2x+3)}$.

b) If $z = \log(e^x + e^y)$ then show that $rt - s^2 = 0$ where
 $r = z_{xx}$, $t = z_{yy}$, $s = z_{xy}$. Symbols have their usual meanings.

c) Verify Euler's theorem for function $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$.

2 Attempt any **TWO** parts of the following :

a) Expand $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in ascending powers of x .

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[Contd...

- b) If u, v, w are roots of equation

$$(x-a)^3 + (x-b)^3 + (x-c)^3 = 0 \quad \text{find} \quad \frac{\partial(u, v, w)}{\partial(a, b, c)}.$$

- c) Using Lagrange's method of Maxima and Minima find the shortest distance from the point $(1, 2, -1)$ to sphere $x^2 + y^2 + z^2 = 24$.

3 Attempt any **TWO** parts of following :

- a) Find the rank of matrix by reducing to Normal form

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}.$$

- b) Find the eigen values of corresponding eigen vectors

$$\text{of matrix } A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

- c) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, find A^{-1} and A^4 using Caylay's

Hamilton's theorem.

4 Attempt any **TWO** parts of following :

- a) Evaluate $\int_0^1 \int_0^1 \sin y^2 dy dx$ by changing the order of integration.

- b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates.
- c) Prove that $\frac{\beta(p, q+1)}{q} = \frac{\beta(p+1, q)}{p} = \frac{\beta(p, q)}{p+q}$
 $(p > 0, q > 0)$.

5 Attempt any **TWO** of following :

- a) Using Green's theorem, evaluate

$$\int_c (x^2 + xy) dx + (x^2 + y^2) dy \text{ where } c \text{ is square}$$

formed by lines $x = \pm 1, y = \pm 1$.

- b) Verify divergence theorem for $\vec{F} = x^3 \hat{i} - y^3 \hat{j} + z^3 \hat{k}$ taken over surface of sphere $x^2 + y^2 + z^2 = a^2$.
- c) If $\vec{F} = (x^2 + yz) \hat{i} + (y^2 + zx) \hat{j} + (z^2 + xy) \hat{k}$ find $div \vec{F}$ and $curl \vec{F}$.