**Printed Pages : 4**

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**AS-101**

(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 199101**

**Roll No.** | | | | | | | | | |

# B. Tech.

## (SEM. I) (ODD SEM.) THEORY EXAMINATION, 2014-15

### MATHEMATICS - I

Time : **3** Hours]                    [Total Marks : **100**

**Note :**    (1)    Attempt **all** questions.
            (2)    All questions carry equal marks.

**1**    Attempt any **two** of following :

(a)    If $y = \left(\sinh^{-1} x\right)^2$, prove that

$$\left[\left(1 + x^2\right) y_{n+2} + \left(2n + 1\right) xy_{n+1} + x^2 y_n = 0\right].$$

(b)    If $u = x \sin^{-1}\left(\dfrac{x}{y}\right) + y \sin^{-1}\left(\dfrac{y}{x}\right)$ find the

value of $\left[x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2}\right]$

(c)  If $u = x^2 + y^2 + z^2 - 2xyz = 1$, show that

$$\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} + \frac{dz}{\sqrt{1-z^2}} = 0 \; .$$

**2**  Attempt any two parts of following :

(a)  Trace the curve $r = a\left(1 + \cos\theta\right)$.

(b)  Find the approximate value of

$$\left[\left(0.98\right)^2 + \left(2.01\right)^2 + \left(1.94\right)^2\right]^{1/2}$$

(c)  If $x = e^u \cos v, \; y = e^u \sin v$ then find

$$\frac{\partial\left(u, v\right)}{\partial\left(x, y\right)} \; .$$

**3**  Attempt any two parts of following :

(a)  Evaluate $\displaystyle\int_0^3 \int_0^{6/x} x^2 \; dy \; dx$ by changing the order

of integration.

(b)  Find the area enclosed by curves $y^2 = 4ax$ and

$x^2 = 4ay$.

(c)  Prove that

$$\beta\left(m, n\right) = a^m b^m \int\limits_{0}^{\infty} \frac{x^{m-1}}{\left(ax + b\right)^{m+n}} \, dx .$$

4    Attempt any **two** parts of following :

(a)  Find $\phi$ if

$$\nabla\phi = \left[\left(y^2 - 2xyz^3\right)\hat{i} + \left(3 + 2xy - x^2z^3\right)\hat{j} + \left(6z^3 - 3x^2yz^2\right)\hat{k}\right]$$

(b)  Evaluate by Green's theorem

$$\oint_c \left[\left(\cos x \sin y - 2xy\right)dx + \sin x \cos y \, dy\right]$$

where  C is circle  $x^2 + y^2 = 1$

(c)  Verify Stoke's theorem for $\vec{F} = \left(y\,\hat{i} + z\,\hat{j} + x\,\hat{k}\right)$

where  S  is  upper  half  surface  of  sphere

$x^2 + y^2 + z^2 = 1$  and  C  is  its  boundary.

5     Attempt any **two** parts of following :

(a)   Find non-singular matrices P and Q such that P
      and Q is in normal form of the matrix and hence
      find the  rank of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}.$$

(b)   Solve following system of equations :

$$2x_1 + 3kx_2 + (3k + 4)x_3 = 0$$
$$x_1 + (k + 4)x_2 + (4k + 2)x_3 = 0$$
$$x_1 + 2(k + 1)x_2 + (3k + 4)x_3 = 0$$

(c)   Find the eigen values and corresponding eigen

vectors  of  matrix  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$

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