Printed pages: 02 **Sub Code: NAS103**

Paper Id: | 9 | 0 | 6 | 0 | **Roll No.** | | | | | | | | | | |

B-Tech
(SEM -I) THEORY EXAMINATION 2017-18
Engineering Mathematics

Time: 3 Hours ***Total Marks: 100***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x10 = 20

(a) Find the n^{th} derivative of x^2e^{-x} $at\ x = 0$.

(b) If $x^2 = au + bv,\ y^2 = au - bv\ \ then\ find\ \frac{\partial u}{\partial x}$.

(c) The Eigen values of a matrix A are 2,3,1 then find the Eigen values of $A^{-1} + A^2$.

(d) If $x = u(1 + v),\ y = v(1 + u)$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

(e) Find Curl $\overline{r}$, $where\ \overline{r} = x\hat{\imath} + y\hat{\jmath} + 2\hat{k}$

(f) Evaluate the area enclosed between the parabola $y = x^2$ and the straight line $y = x$.

(g) Find the Tyalor's series expansion of : $f(x, y) = x^3 + xy^2$ about point (2,1).

(h) If $f(x, y, z, w) = 0$ Find the value $\frac{\partial x}{\partial y} \times \frac{\partial y}{\partial z} \times \frac{\partial z}{\partial w} \times \frac{\partial w}{\partial x}$.

(i) Evaluate $\int_0^\infty e^{-x^2}\ dx$

(j) Write the statement of divergence theorem for a given vector field $\overrightarrow{F}$.

SECTION B

2. Attempt any *three* of the following: 10 x 3 = 30

(a) If $y = \sin(a \sin^{-1} x)$, find $y_n(0)$.

(b) Verify Euler's theorem for $= \frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}$.

(c) Find the Eigen values and corresponding Eigen vectors of $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(d) Compute $\iiint_v x^2\ dx\ dy\ dz$ over volume of tetrahedron bounded by $x = 0$, $y = 0,\ z = 0\ \ and\ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$..

(e) Verify the Green's theorem to evaluate the line integral $\int_C (2y^2 dx + 3x\ dy)$, where C is the boundary of the closed region bounded by $y = x\ and\ y = x^2$.

SECTION C

3. Attempt any *one* part of the following: 10 x 1 = 10

(a) If $x = \sin\sqrt{y},\ prove\ that\ (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$.

(b) If $u = sec^{-1}\left(\frac{x^3-y^3}{x+y}\right), find\ x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$.

4. Attempt any *one* part of the following: 10 x 1 = 10

(a) A balloon is in the form of right circular cylinder of radius 1.5m and length 4m and is surmounted by hemispherical ends. If the radius is increased by 0.01m and the length by 0.05m find the percentage change in the volume of the balloon.

(b) Show that the function: $u = x + y + z, v = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are functionally related. Find the relation between them.

5. Attempt any *one* part of the following: 10 x 1 = 10

(a) Test the consistency and hence, solve the following set of equations
10y + 3z = 0; 3x + 3y + z = 1;
2x – 3y – z = 5; x + 2y = 4.

(b) Reduce the matrix in to normal form and hence find its rank

$$\begin{bmatrix} 0 & 1 & 2 & -1 \\ 1 & 2 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

6. Attempt any *one* part of the following: 10 x 1 = 10

(a) Change the order of Integration in $I = \int_0^2 \int_{\frac{x^2}{4}}^{3-x} xy\ dxdy$ and hence evaluate it.

(b) Define Beta and Gamma function and Evaluate $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{\frac{1}{2}} dx$.

7. Attempt any *one* part of the following: 10 x 1 = 10

(a) Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$, where $r = |\vec{r}|$ is irrotational. Find the scalar potential.

(b) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ taken round the rectangle bounded by the lines $x = \pm a;\ \ y = 0,\ y = b$.