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NAS103/EAS103

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199125

Roll No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

B.Tech. (Semester-I)

SPL. THEORY EXAMINATION, 2014-15

ENGINEERING MATHEMATICS-I

Time : 3 Hours] ***[Total Marks : 100***

Section – A

Attempt all parts of this question. Each part carries two marks. 2×10=20

1. (a) If $y = \log x^3$, then find y_n.

(b) If $\log\left(x^3 + y^3 - x^2 y - xy^2\right)$ then $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$

(c) If $f(x) = f(0) + k f_1(0) + \frac{k^2}{2!} f_2(\theta k);\ 0 < \theta < 1$, then find the value of θ when $k = 1$ and $f(x) = (1-x)^{5/2}$.

(d) Find the percentage error in the area of an ellipse when an error of +1% is made in measuring the major and minor axes.

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(e) Find the value of 'b' for which the rank of the matrix

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$$ is 2.

(f) Show that the vectors $X_1 = [1,\ 2,\ 3]$, $X_2 = [2,\ -2,\ 0]$ form a linearly independent Set.

(g) Evaluate $\iint_D (x^2 + y^2)\, dx\, dy$ where D is bounded by $y = x$ and $y^2 = 4x$.

(h) Find the value of $\Gamma\left(-\frac{5}{2}\right)$.

(i) Prove that if $\vec{u}$ and $\vec{v}$ are irrotational then $\vec{u} \times \vec{v}$ is solenoidal.

(j) Write the statement of Gauss divergence theorem.

Section – B

Attempt any three parts of the following: 3×10=30

2. (a) If $y = \left(\sinh^{-1} x\right)^2$, then find $(y_n)_0$.

(b) Find the values of a and b such that expansion of $\log(1+x) - \frac{x(1+ax)}{(1+bx)}$ in ascending powers of x begins with the terms x^4 and hence find this term.

(c) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, –1, 2).

—x—

(c) Find the Eigen-values and Eigen-vectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

(d) Evaluate $\iint y\,dx\,dy$ over the part of the plane bounded by the line $y=x$ and the parabola $y=4x-x^2$.

(e) Use divergence theorem to evaluate the surface integral $\iint_S x\,dy\,dz+y\,dz\,dx+z\,dx\,dy$ where S is the portion of the plane $x+2y+3z=6$ which lies in the first octant.

Section – C

Attempt all questions of this section. Attempt any two parts from each question. Each question carries 10 marks. (2×5)×5=50

3. (a) If $y^{1/m}+y^{-1/m}=2x$, prove that:

$$\left(x^2-1\right)y_{n+2}+\left(2n+1\right)xy_{n+1}+\left(n^2-m^2\right)y_n=0$$

(b) Find $\frac{dy}{dx}$ when $(\cos x)^y=(\sin y)^x$.

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(c) Find the value of $\frac{\partial^2 y}{\partial x^2}$ by changing the independent variable x to z, by the substitution $x = \tan z$.

4. (a) If $x = \sqrt{vw}$, $y = \sqrt{uw}$, $z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi$, $v = r\sin\theta\sin\phi$, $w = r\cos\theta$, then calculate the Jacobian $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$.

(b) If Δ is the area of a triangle, prove that the error in Δ resulting from a small error in c is given by
$$\delta\Delta = \frac{\Delta}{4}\left[\frac{1}{s} + \frac{1}{s-a} + \frac{1}{s-b} - \frac{1}{s-c}\right]\delta c.$$

(c) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

5. (a) Show that the vectors (3, 0, 2), (7, 0, 9) and (4, 1, 2) form a basis for E^3.

(b) Find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & -3 \\ 1 & 1 & 2 & 3 \end{bmatrix}$ by reducing it to normal form.

(c) Show that the equations $-2x + y + z = a$, $x - 2y + z = b$, $x + y - 2z = c$ have no solution unless $a + b + c = 0$. In which case they have infinitely many solutions? Find these solutions when $a = 1$, $b = 1$, $c = -2$.

6. (a) Change the order of integration in the following integral and evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dydx$.

(b) Evaluate $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dydxdz$.

(c) Evaluate $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{1/2} dx$.

7. (a) Find $grad\phi$ when ϕ is given by $\phi = 3x^2y - y^3z^2$ at the point (1, –2, –1).

(b) Prove that $\vec{a} \times \left(\vec{\nabla} \times \vec{r}\right) = \vec{\nabla}\left(\vec{a} \cdot \vec{r}\right) - \left(\vec{a} \cdot \vec{\nabla}\right)\vec{r}$ where $\vec{a}$ is constant vector.