

**B. TECH**  
**(SEM I) THEORY EXAMINATION 2019-20**  
**ENGINEERING MATHEMATICS-I**

Time: 3 Hours

Total Marks: 70

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

**SECTION A**

1. Attempt all questions in brief. 2 x 7 = 14

a.	For what value of 'λ', the vectors (1, -2, λ), (2, -1, 5) and (3, -5, 7λ) are linearly dependent.
b.	If A is a skew-Hermitian matrix, then show that iA is Hermitian.
c.	Find the maximum value of the function $f(x, y, z) = (z - 2x^2 - 2y^2)$ where $3xy - z + 7 = 0$ .
d.	Evaluate $\int_0^1 \int_0^{x^2} e^{y/x} dy dx$
e.	Show that the vector field $\vec{V} = (\sin y + z)\hat{i} + (x \cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational.
f.	Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.
g.	For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$ , find the magnitude of gradient at the point(1,3)

**SECTION B**

2. Attempt any three of the following: 7 x 3 = 21

a.	(i) Express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A where $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ . <span style="float: right;">(3)</span>  (ii) Reduce the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ to diagonal form. <span style="float: right;">(4)</span>
b.	(i) Trace the curve $r = 2a \cos \theta$ . <span style="float: right;">(3)</span>  (ii) If $u = \cos ec^{-1} \left( \frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}} \right)^{1/2}$ prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u)$ . <span style="float: right;">(4)</span>
c.	(i) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. <span style="float: right;">(3)</span>  (ii) If $u = x^3 + x^2y + x^2z - z^2(x + y + z)$ , $v = x + z$ , $w = x^2 - z^2 + xy - zy$ Prove that $u, v$ and $w$ are connected by a functional relation <span style="float: right;">(4)</span>
d.	(i) Change the order of integration for $I = \int_0^1 \int_{x^2}^{2-x} xy dx dy$ and hence evaluate the same. <span style="float: right;">(3)</span>  (ii) A triangular prism is formed by planes whose equations are $ay = bx, y = 0$ and $x = a$ . Find the volume of the prism between the planes $z = 0$ and surface $z = c + xy$ . <span style="float: right;">(4)</span>

e.	<p>(i) If <math>\vec{F} = 3xy\hat{i} - y^2\hat{j}</math>, evaluate <math>\int_C \vec{F} \cdot d\vec{r}</math>, where C is the arc of the parabola <math>y = 2x^2</math> from (0,0) to (1,2). <span style="float: right;">(3)</span></p> <p>(ii) Find the directional derivative of <math>\frac{1}{r}</math> in the direction of <math>\vec{r}</math> where <math>\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}</math>. <span style="float: right;">(4)</span></p>
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**SECTION C**

**3. Attempt any one part of the following: 7 x 1 = 7**

(a)	If $y = e^{m \sin^{-1} x}$ , prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ and calculate $y_n(0)$ .
(b)	If $x = e^r \cos \theta, y = e^r \sin \theta$ , show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2r} \left( \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2} \right)$ .

**4. Attempt any one part of the following: 7 x 1 = 7**

(a)	If $f(x, y) = x^2 y^{1/10}$ , compute the value of $f$ when $x = 1.99$ and $y = 3.01$ .
(b)	A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

**5. Attempt any one part of the following: 7 x 1 = 7**

(a)	Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
(b)	Find non-singular matrices P and Q such that PAQ is in normal form of the matrix and hence find the rank of matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ .

**6. Attempt any one part of the following: 7 x 1 = 7**

(a)	Find the volume of the portion of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which lies in the first octant.
(b)	Transform the double integral $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dydx}{\sqrt{a^2-x^2-y^2}}$ into polar coordinates and then evaluate it.

**7. Attempt any one part of the following: 7 x 1 = 7**

(a)	Verify Gauss divergence theorem for $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ over the parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0, z = c$ .
(b)	Evaluate the integral $\int_C \{(x+y)dx + (2x-z)dy + (y+z)dz\}$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6) using stoke's theorem.