 Sub Code:NAS103

Paper Id: 199123 **Roll No:**

B. TECH.
(SEM I) THEORY EXAMINATION 2019-20
ENGINEERING MATHEMATICS -I

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

a.	Find y_n, if $y = x^2 \log x$.
b.	If $u(x,y) = \left(\sqrt{x} + \sqrt{y}\right)^{\frac{1}{5}}$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.
c.	Calculate $\frac{\partial(u,v)}{\partial(x,y)}$ for $x = e^u \cos v$, and $y = e^u \sin v$.
d.	Prove that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots\ldots\ldots\ldots\ldots$
e.	Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.
f.	Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.
g.	Evaluate $\int_0^1 x^2 (1-x)^3 \, dx$.
h.	Evaluate $\int_0^1 \int_0^{x^2} x \, dy dx$.
i.	Show that $\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ is irrotational.
j.	State Gauss divergence theorem.

SECTION B

2. Attempt any *three* of the following: 10x3=30

a.	If $y = (\sin^{-1} x)^2$, show that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$ and calculate $y_n(0)$.
b.	Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
c.	Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ $to\ the\ diagonal\ form.$
d.	Find the volume of the solid surrounded by the surface $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} + \left(\frac{z}{c}\right)^{\frac{2}{3}} = 1$.
e.	Verify Stokes's theorem for $\vec{F} = x^2\hat{\imath} + xy\hat{\jmath}$ integrated round the square whose sides are $x = 0, y = 0, x = a, y = a$ in the plane $z = 0$.

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SECTION C

3. Attempt any *one* part of the following: 10x1=10

a.	Trace the curve: $y^2(2a-x)=x^3$
b.	If $u=\log(x^3+y^3+z^3-3xyz)$, show that $(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z})^2 u=-\frac{9}{(x+y+z)^2}$.

4. Attempt any *one* part of the following: 10x1=10

a.	Expand $\tan^{-1}\frac{y}{x}$ in the neighbourhood of $(1,1)$ upto and inclusive of second-degree terms. Hence compute $f(1.1,0.9)$ approximately.
b.	If u,v,w are the roots of the equation $(x-a)^3+(x-b)^3+(x-c)^3=0$, then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$.

5. Attempt any *one* part of the following: 10x1=10

a.	Find the value of λ such that the following equations have unique solution: $\lambda x+2y-2z-1=0, 4x+2\lambda y-z-2=0, 6x+6y+\lambda z-3=0$ and use matrix method to solve these equations when $\lambda=2$.
b.	Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 2 & 0\\ 2 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$ and hence find A^{-1}

6. Attempt any *one* part of the following: 10x1=10

a.	Show that in the Catenary $y=c\cosh\frac{x}{c}$, the length of arc from the vertex $x=0$ to any point (x,y) is given by $s=c\sinh\frac{x}{c}$.
b.	Evaluate $\int_0^a\int_y^a \frac{x\,dxdy}{x^2+y^2}$ by changing the order of integration.

7. Attempt any *one* part of the following: 10x1=10

a.	Find the directional derivative of $\emptyset=5x^2y-5y^2z+\frac{5}{2}z^2x$ at the point $P(1,1,1)$ in the direction of the line $\frac{x-1}{2}=\frac{y-3}{-2}=\frac{z}{1}$.
b.	Apply Green's theorem to evaluate $\int_C [(2x^2-y^2)dx+(x^2+y^2)dy]$ where C is the boundary of the area enclosed by the $x-axis$ and upper half of the circle $x^2+y^2=a^2$.