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Paper ID:

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Roll No.

B. TECH.
(SEM. I) THEORY EXAMINATION 2018-19
ENGINEERING MATHEMATICS -I

Time: 3 Hours ***Total Marks: 100***

Note: 1. Attempt all sections. If require any missing data; then choose suitably.

SECTION – A

1. Attempt **ALL** the parts: **(2x10 = 20)**

(a) Find y_n, if $y = sin^2 x$ at $x = 0$.

(b) If $u(x,y) = \left(\sqrt{x} + \sqrt{y}\right)^5$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x\, \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.

(c) Calculate $\frac{\partial(u,v)}{\partial(x,y)}$ for $x = e^u \cos v$, and $y = e^u \sin v$.

(d) Prove that $f(ax) = f(x) + (a-1)xf'(x) + \frac{(a-1)^2 x^2}{2!} f''(x) + \frac{(a-1)^3 x^3}{3!} f'''(x) + \cdots \ldots \ldots \ldots$

(e) Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into normal form.

(f) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.

(g) Find the value of $\int_0^\infty x^{\frac{1}{4}} e^{-\sqrt{x}} dx$.

(h) Evaluate $\int_0^1 \int_0^{x^2} x e^y \, dy dx$.

(i) Show that $\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ is irrotational.

(j) State Gauss divergence theorem.

SECTION – B

2. Attempt any **THREE** of the following: **(10x3=30)**

(a) If $y = \sin(a \sin^{-1} x)$, show that
$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 - a^2) y_n = 0$ and calculate $y_n(0)$.

(b) Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(c) Diagonalise the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(d) Find the volume of the solid surrounded by the surface $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} + \left(\frac{z}{c}\right)^{\frac{2}{3}} = 1$.

(e) Verify Stoke's theorem for $\vec{F} = x^2 \hat{\imath} + xy\hat{\jmath}$ integrated round the square whose sides are $x = 0, y = 0, x = a, y = a$ in the plane $z = 0$.

SECTION – C

3. Attempt any **TWO** of the following: **(5x2=10)**

(a) Verify Euler's theorem for the function: $u = \frac{x^2y^2}{x+y}$

(b) If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

(c) Trace the curve: $y^2 = x^3$.

4. Attempt any **TWO** of the following: **(5x2=10)**

(a) Expand $\tan^{-1}\frac{y}{x}$ in powers of $(x - 1)$ and $(y - 1)$ upto two degree terms.

(b) Show that $u = \frac{x+y}{z}, v = \frac{y+z}{x}, w = \frac{y(x+y+z)}{xz}$ are not independent, find the relation among them.

(c) Find the extreme values of $3x^2 - y^2 + x^3$.

5. Attempt any **TWO** of the following: **(5x2=10)**

(a) Show that the system of equations:
$x + 3y - 2z = 0, 2x - y + 4z = 0, x - 11y + 14z = 0$ has a non-trivial solution.

(b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and hence find A^{-1}.

(c) Prove that the each characteristic roots of a unitary matrix are of unit modulus.

6. Attempt any **TWO** of the following: **(5x2=10)**

(a) Evaluate $\int_0^a \int_y^a \frac{x\,dxdy}{x^2+y^2}$ by changing the order of integration.

(b) Prove that $\int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{1}{4\sqrt{2}} B\left(\frac{1}{4}, \frac{1}{2}\right)$.

(c) Show that in the Catenary $y = c\cosh\frac{x}{c}$, the length of arc from the vertex $x = 0$ to any point (x, y) is given by $s = c\sinh\frac{x}{c}$.

7. Attempt any **TWO** of the following: **(5x2=10)**

(a) Find the directional derivative of $\emptyset = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point $P(1,1,1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.

(b) Prove that $div(\vec{a} \times \vec{b}) = \vec{b}.curl\,\vec{a} - \vec{a}.curl\,\vec{b}$.

(c) Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the area enclosed by the $x - axis$ and upper half of the circle $x^2 + y^2 = a^2$.

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