

B.TECH
(SEM - I) THEORY EXAMINATION 2018-19
ENGINEERING MATHEMATICS - I

Time: 3 Hours

Total Marks: 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

- a. If $u(x, y) = (\sqrt{x} + \sqrt{y})^5$, find value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.
- b. Find all symmetry of the curve $y^2(2a - x) = x^3$.
- c. If $x = r \cos \theta, y = r \sin \theta$ find value of $\frac{\partial(x, y)}{\partial(r, \theta)}$.
- d. Find the percentage error in the area of an ellipse when an error of +1% is made in measuring the major and minor axes.
- e. Find rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$.
- f. Show that the matrix $\begin{bmatrix} 2 & 3-4i \\ 3+4i & 5 \end{bmatrix}$ is Hermitian.
- g. Change the order of integration in $\int_0^{\infty} \int_x^{\infty} f(x, y) dy dx$.
- h. Use triple integral to find volume of a sphere of unit radius.
- i. Find $\text{grad} \phi$ when ϕ is given by $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, -1)$.
- j. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ find $\text{div} \vec{r}$ and $\text{curl} \vec{r}$.

SECTION B

2. Attempt any *three* of the following: 10 x 3 = 30

- a. If $y = e^{a \sin^{-1} x}$, find $(y_n)_0$.
- b. If $u^3 + v^3 + w^3 = x + y + z, u^2 + v^2 + w^2 = x^2 + y^2 + z^2$ and $u + v + w = x^2 + y^2 + z^2$, then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.
- c. Diagonalise the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$.
- d. Evaluate $\iint_R (x^2 + y^2) dx dy$ where R is the region in the first quadrant bounded by $x^2 - y^2 = a, x^2 - y^2 = b, 2xy = c, 2xy = d, 0 < a < b, 0 < c < d$.

- e. Verify Green's theorem for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by the lines $x = 0, y = 0, x + y = 1$.

SECTION C

3. **Attempt any one part of the following:** **10 x 1 = 10**
- (a) Expand $\tan^{-1} \frac{y}{x}$ in the neighbourhood of (1,1) upto and inclusive of second degree terms. Hence compute $f(1.1, 0.9)$ approximately.
- (b) Trace the curve $r = a(1 + \cos \theta)$.
4. **Attempt any one part of the following:** **10 x 1 = 10**
- (a) In estimating the number of bricks in a pile which is measured to be $(5m \times 10m \times 5m)$, the count of bricks is taken as 100 bricks per m^3 . Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of the bricks is Rs. 2000 per thousand bricks.
- (b) Find the volume of largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
5. **Attempt any one part of the following:** **10 x 1 = 10**
- (a) Determine the values of λ and μ such that the system $2x - 5y + 2z = 8,$
 $2x + 4y + 6z = 5, , x + 2y + \lambda z = \mu$.
- (b) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, Hence compute A^{-1} .
6. **Attempt any one part of the following:** **10 x 1 = 10**
- (a) Determine the area of region bounded by the curves $xy = 2, 4y = x^2, y = 4$.
- (b) Find the volume of the solid surmounted by the surface $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} + \left(\frac{z}{c}\right)^{2/3} = 1$.
7. **Attempt any one part of the following:** **10 x 1 = 10**
- (a) Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal.
- (b) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.