(Following paper ID and Roll No. to be filled in your answer book)
PAPER ID: **Roll No.**

B.Tech

(SEM. I) THEORY EXAMINATION 2014-15

ENGINEERING MATHEMATICS – I (EAG101/NAG 101)

Time : 3 Hours ***Total Marks : 100***

1. Attempt any four parts of the following:

a) Define the continuity? Consider the function f(x) = $\begin{cases} 2x^2 + 3 \; if \; x \leq 3 \\ 3x + B \; if \; x > 3 \end{cases}$

Find a value of B such that f(x) is continuous at x = 3.

b) Find the limit of f(x) as x tends to 2 from the left if f(x) = $\begin{cases} x^3 - 2 \; if \; x \geq 2 \\ 1 + x^2 \; if \; x < 2 \end{cases}$

c) Determine the co-ordinates of the stationary points of each of the following functions $x = 2t^3 + 1$, $y = te^{-2t}$

d) For each of the following functions determine $\frac{d^2y}{dx^2}$ in terms of t.

x = sin t, y = cos t

e) Find $\int \frac{x+3}{x^2+2x+5} dx$.

f) Find $\int e^x \cot(e^x) dx$.

2. Attempt any two parts of the following:

a) Apply Maclaurin's theorem to find the expansion in ascending powers of x of $\log_e (1+e^x)$ to terms containing x^4.

b) If u= f(r) where $r^2 = x^2+y^2$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

c) If $u_1 = x_2x_3 / x_1$, $u_2 = x_3 x_1 / x_2$, $u_3 = x_1 x_2 / x_3$, prove that $J(u_1, u_2, u_3) = 4$.

3. Attempt any two parts of the following:

a) Find by double integration the area lying between the parabola $y = 4x - x^2$ and the line y = x.

b) Evaluate $\iiint z^2 \; dx \, dy \, dz$ over the sphere $x^2 + y^2 + z^2 = 1$.

c) Define the Beta function, prove that B(m, n) = B(m + 1,n) + B(m,n+1) where m, n > 0.

4. Attempt any four parts of the following:

a) Solve $(3y - 2xy^3)dx + (4x - 3x^2 y^2)dy = 0$

b) Solve the following differential equation.

$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 6y = \sin 3x + \cos 2x$

c) Solve $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = e^{ax} + e^{bx}$

d) Solve the following system of differential equations.

Dx + Dy + 3x = sin t

Dx + y – x = cos t

e) Solve by changing the independent variable.

$(1 + x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$

f) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$$

5. Attempt any two parts of the following:

a) Reduce the matrix A = $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 3 & 1 & 1 & 3 \end{bmatrix}$ to its normal form and hence determine its rank.

b) Find the eigen values and corresponding eigen vectors of the given matrix.

A = $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

c) Find the characteristic equation of matrix A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1}.