Printed Pages : 4

AG-101

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 992101

Roll No.

B.Tech.

(SEM. I) (ODD SEM.) THEORY EXAMINATION, 2014-15

ENGINEERING MATHEMATICS - I

Time : **3** Hours] [Total Marks : **100**

Section - A

1. Attempt all parts of this question. Each part carries two marks **2×10=20**

(a) Find the derivative of $=\cos\left(\sqrt{x}\right)$

(b) Evaluate: $\int \log x \, dx$

(c) If u = $e^x \sin y$ and $v = e^x \cos y$, Evaluate : $\frac{\partial(u,v)}{\partial(x,y)}$.

(d) $u = x^y$, then $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

(e) Prove that $\Gamma\left(-\frac{3}{2}\right) = \frac{4}{3}\sqrt{\pi}$

(f) Evaluate: $\beta(1,2) + \beta(2,1)$

(g) What is the difference between general solution and particular solution of a differential equation?

(h) Find the order and degree of the differential equation $y'''+2y''+y'=0$

(i) For the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ find the sum of the Eigen values.

(j) State Cayley-Hamilton theorem.

Section - B

2. Attempt any three parts of this question. **3×10=30**

(a) Evaluate $\int \frac{3x-2}{(x+1)^2(x+3)}dx$.

(b) If $u=\log(x^3+y^3+z^3-3xyz)$, show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$$

(c) Evaluate: $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\,dxdy$

(d) Find the Particular Solution of differential equation $\frac{dy}{dx}+y\cot x=2x+x^2$ given that $y=0\,when\,x=\frac{\pi}{2}$

(e) Test whether the following systems of equations possess a non-trivial solution:

$x_1+x_2+2x_3+3x_4=0; 3x_1+4x_2+7x_3+10x_4=0;$

$5x_1+7x_2+11x_3+17x_4=0; 6x_1+8x_2+13x_3+16x_4=0$

Section - C

Attempt any two parts from each question. All questions are compulsory. **(5×2)×5=50**

3. (a) Find the derivative of $x^2 \sin x$ with respect to x.

(b) Find the all point of discontinuity $f(x)=\begin{cases} 2x+3\ , x\le 2 \\ 2x-3\ , x>2 \end{cases}$

(c) Integrate: $\int \frac{e^{\tan^{-2x}}}{1+x^2}dx$

4. (a) If $\log\left(x^3+y^3-x^2y-xy^2\right)$ then $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}$.

(b) What is the degree of homogenous function $u(x,y)=\frac{x^2\left(x^2-y^2\right)^{1/3}}{\left(x^2+y^2\right)^{2/3}}$?

(c) Find the value of $\frac{\partial^2 y}{\partial x^2}$ by changing the independent variable x to y, by the substitution $z=\frac{1}{x}$.

5. (a) Evaluate the following integral by changing the order of integration $\int_0^1 \int_{e^x}^{e} \frac{dydx}{\log y}$.

(b) Evaluate $\int\int xydxdy$ over the positive quadrant of the circle $x^2+y^2+a^2$.

(c) Find the area lying between parabola $y=4x-x^2$ and the line $y = x$.

6. (a) Solve: $\frac{dy}{dx}=-4xy^2$

(b) Solve: $(D^2-4D+4)\,y=x^2e^{2x}$

(c) Using variation of parameter method to solve : $\frac{d^2y}{dx^2}+y=\sec x$

7. (a) Find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by elementary transformation.

(b) Under what condition, the rank of the following matrix A is 3. Is it possible for the rank to be 1 ? Why

$$A=\begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$$

(c) Find the Eigen values of the following matrix $\begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$